Solutions Manual

6th Edition

Feedback Control of Dynamic Systems

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Chapter 5

The Root-Locus Design Method

Problems and solutions for Section 5.1

1. Set up the following characteristic equations in the form suited to Evans's root-locus method. Give $L(s)$, $a(s)$, and $b(s)$ and the parameter, K, in terms of the original parameters in each case. Be sure to select K so that $a(s)$ and $b(s)$ are monic in each case and the degree of $b(s)$ is not greater than that of $a(s)$.

   (a) $s + (1/\tau) = 0$ versus parameter τ

   (b) $s^2 + cs + c + 1 = 0$ versus parameter c

   (c) $(s + c)^3 + A(Ts + 1) = 0$

        i. versus parameter A,

        ii. versus parameter T,

        iii. versus the parameter c, if possible. Say why you can or can not. Can a plot of the roots be drawn versus c for given constant values of A and T by any means at all

   (d) $1 + [k_p + \dfrac{k_I}{s} + \dfrac{k_D s}{\tau s + 1}]G(s) = 0$. Assume that $G(s) = A\dfrac{c(s)}{d(s)}$ where $c(s)$ and $d(s)$ are monic polynomials with the degree of $d(s)$ greater than that of $c(s)$.

        i. versus k_p

        ii. versus k_I

        iii. versus k_D

        iv. versus τ

Solution:

(a) $K = 1/\tau$; $a = s$; $b = 1$

(b) $K = c$; $a = s^2 + 1$; $b = s + 1$

(c) Part (c)

    i. $K = AT$; $a = (s + c)^3$; $b = s + 1/T$

    ii. $K = AT$; $a = (s + c)^3 + A$; $b = s$

    iii. The parameter c enters the equation in a nonlinear way and a standard root locus does not apply. However, using a polynomial solver, the roots can be plotted versus c.

(d) Part (d)

    i. $K = k_p A\tau$; $a = s(s + 1/\tau)d(s) + k_I(s + 1/\tau)c(s) + \dfrac{k_D}{\tau}s^2 Ac(s)$; $b = s(s + 1/\tau)c(s)$

    ii. $K = Ak_I$; $a = s(s + 1/\tau)d(s) + Ak_p s(s + 1/\tau) + \dfrac{k_D}{\tau}s^2 Ac(s)$; $b = s(s + 1/\tau)c(s)$

    iii. $K = \dfrac{Ak_D}{\tau}$; $a = s(s + 1/\tau)d(s) + Ak_p s(s + 1/\tau)c(s) + Ak_I(s + 1/\tau)c(s)$; $b = s^2 c(s)$

    iv. $K = 1/\tau$; $a = s^2 d(s) + k_p As^2 c(s) + k_I Asc(s)$; $b = sd(s) + k_p sAc(s) + k_I Ac(s) + k_D s^2 Ac(s)$

Problems and solutions for Section 5.2

2. Roughly sketch the root loci for the pole-zero maps as shown in Fig. 5.51. Show your estimates of the center and angles of the asymptotes, a rough evaluation of arrival and departure angles for complex poles and zeros, and the loci for positive values of the parameter K. Each pole-zero map is from a characteristic equation of the form

$$1 + K\frac{b(s)}{a(s)} = 0,$$

where the roots of the numerator $b(s)$ are shown as small circles o and the roots of the denominator $a(s)$ are shown as $\times's$ on the s-plane. Note that in Fig. 5.51(c), there are two poles at the origin and there are two poles on the imaginary axis in (f), slightly off the real axis.

Solution:

Root loci for Problem 5.2
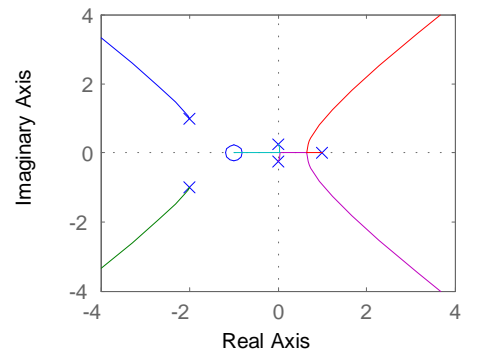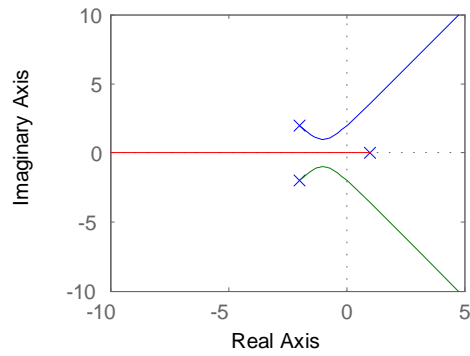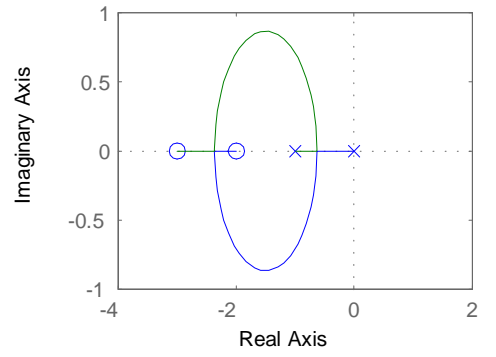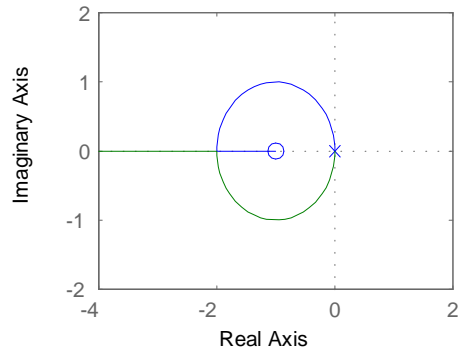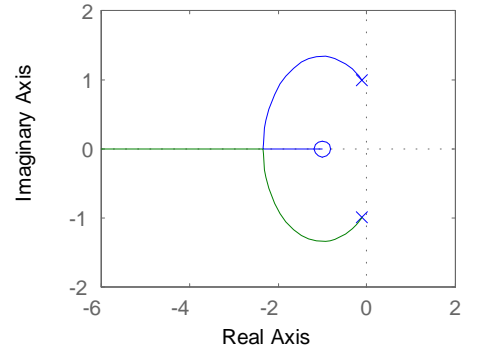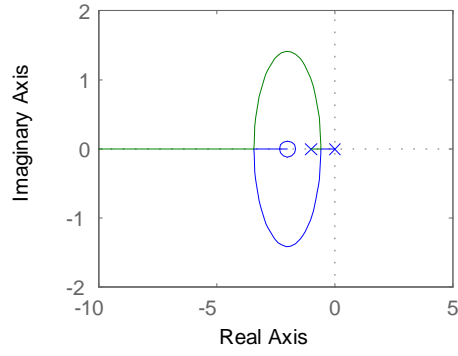
We had to make up some numbers to do it on Matlab, so the results partly depend on what was dreamed up, but the idea here is just get the basic rules right.

(a)$a(s) = s^2 + s$; $b(s) = s + 1$

Breakin(s)   -3.43; Breakaway(s)   -0.586

(b) $a(s) = s^2 + 0.2s + 1$; $b(s) = s + 1$

Angle of departure: 135.7

Breakin(s)   -4.97

(c) $a(s) = s^2$; $b(s) = (s + 1)$

Breakin(s)   -2

(d) $a(s) = s^2 + 5s + 6$; $b(s) = s^2 + s$

Breakin(s)   -2.37

Breakaway(s)   -0.634

(e) $a(s) = s^3 + 3s^2 + 4s - 8$

Center of asymptotes -1

Angles of asymptotes ± 60, 180

Angle of departure: -56.3

(f) $a(s) = s^3 + 3s^2 + s - 5$; $b(s) = s + 1$

Center of asymptotes -.667

Angles of asymptotes ± 60, -180

Angle of departure: -90

Breakin(s)   -2.06

Breakaway(s)   0.503

But, to get this one right, all you have to do is get the real axis segments and the 4 asymptotes going out at the right angles.   You don't know, really, where it breaks away from the real axis nor where the center of asymptotes are.
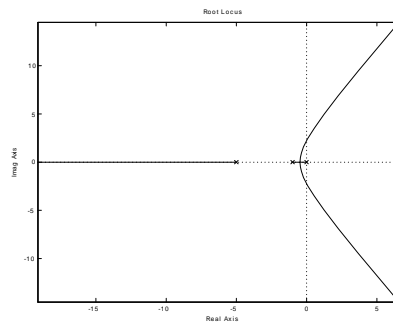
3. For the characteristic equation

$$1 + \frac{K}{s(s+1)(s+5)} = 0 :$$

(a) Draw the real-axis segments of the corresponding root locus.

(b) Sketch the asymptotes of the locus for $K \to \infty$.

(c) For what value of K are the roots on the imaginary axis?

(d) Verify your sketch with a MATLAB plot.

Solution:

(a) The real axis segments are $0 > \sigma > -1$; $-5 > \sigma$

(b) $\alpha = -6/3 = -2$; $\phi_i = \pm 60$, 180

(c) $K_o = 30$

Solution for Problem 5.3

4. *Real poles and zeros.*   Sketch the root locus with respect to K for the equation $1 + KL(s) = 0$ and the following choices for $L(s)$. Be sure to give the asymptotes, arrival and departure angles at any complex zero or pole, and the frequency of any imaginary-axis crossing. After completing each hand sketch verify your results using MATLAB. Turn in your hand sketches and the MATLAB results on the same scales.

(a) $L(s) = \dfrac{(s+2)}{s(s+1)(s+5)(s+10)}$

(b) $L(s) = \dfrac{1}{s(s+1)(s+5)(s+10)}$

(c) $L(s) = \dfrac{(s+2)(s+6)}{s(s+1)(s+5)(s+10)}$

(d) $L(s) = \dfrac{(s+2)(s+4)}{s(s+1)(s+5)(s+10)}$
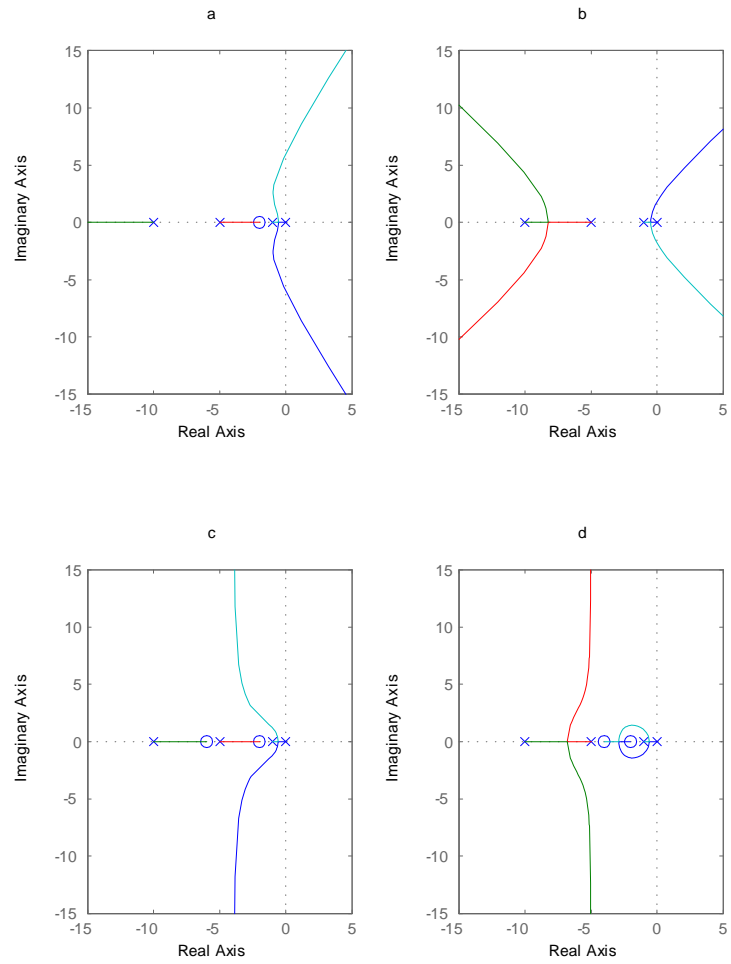
Solution:

All the root locus plots are displayed at the end of the solution set for this problem.

(a) $\alpha = -4.67$; $\phi_i = \pm 60$; ± 180; $\omega_o = 5.98$

(b) $\alpha = -4$; $\phi_i = \pm 45$; ± 135; $\omega_o = 1.77$

(c) $\alpha = -4$; $\phi_i = \pm 90$; $\omega_o - > none$

(d) $\alpha = -5$; $\phi_i = \pm 90$; $\omega_o - > none$

Root loci for Problem 5.4

5. *Complex poles and zeros* Sketch the root locus with respect to K for the equation $1 + KL(s) = 0$ and the following choices for $L(s)$. Be sure to give the asymptotes, arrival and departure angles at any complex zero or pole, and the frequency of any imaginary-axis crossing. After completing each hand sketch verify your results using MATLAB. Turn in your hand sketches and the MATLAB results on the same scales.

(a) $L(s) = \dfrac{1}{s^2 + 3s + 10}$

(b) $L(s) = \dfrac{1}{s(s^2 + 3s + 10)}$

(c) $L(s) = \dfrac{(s^2 + 2s + 8)}{s(s^2 + 2s + 10)}$

(d) $L(s) = \dfrac{(s^2 + 2s + 12)}{s(s^2 + 2s + 10)}$

(e) $L(s) = \dfrac{(s^2 + 1)}{s(s^2 + 4)}$

(f) $L(s) = \dfrac{(s^2 + 4)}{s(s^2 + 1)}$

Solution:

All the root locus plots are displayed at the end of the solution set for this problem.

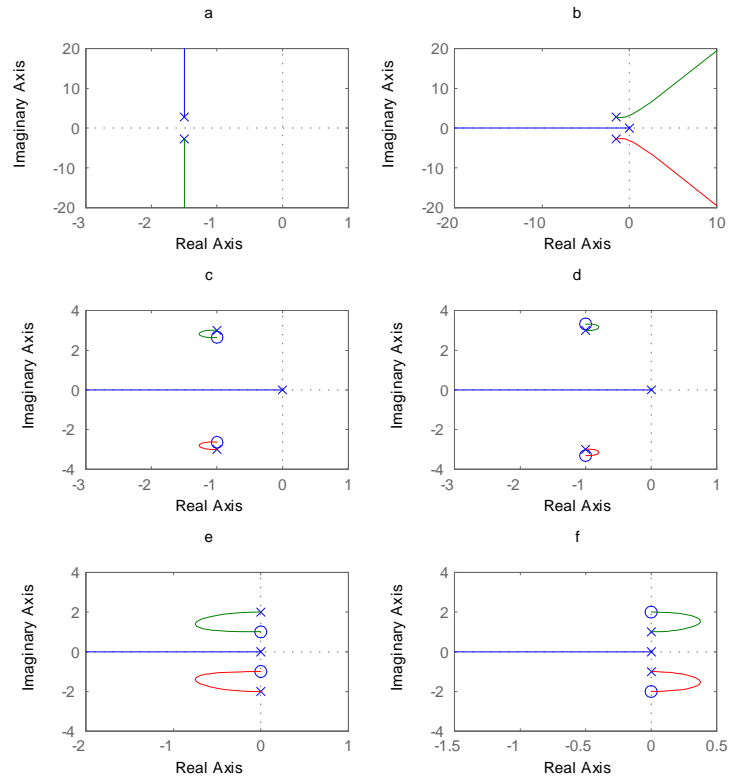(a) $\alpha = -3$; $\phi_i = \pm 90$; $\theta_d = \pm 90$ $\omega_o - > none$

(b) $\alpha = -3$; $\phi_i = \pm 60, \pm 180$; $\theta_d = \pm 28.3$ $\omega_o = 3.16$

(c) $\alpha = -2$; $\phi_i = \pm 180$; $\theta_d = \pm 161.6$; $\theta_a = \pm 200.7$; $\omega_o - > none$

(d) $\alpha = -2$; $\phi_i = \pm 180$; $\theta_d = \pm 18.4$; $\theta_a = \pm 16.8$; $\omega_o - > none$

(e) $\alpha = 0$; $\phi_i = \pm 180$; $\theta_d = \pm 180$; $\theta_a = \pm 180$; $\omega_o - > none$

(f) $\alpha = 0$; $\phi_i = \pm 180$; $\theta_d = 0$; $\theta_a = 0$; $\omega_o - > none$

Root loci for Problem 5.5

6. *Multiple poles at the origin* Sketch the root locus with respect to K for the equation $1 + KL(s) = 0$ and the following choices for $L(s)$. Be sure to give the asymptotes, arrival and departure angles at any complex zero or pole, and the frequency of any imaginary-axis crossing. After completing each hand sketch verify your results using MATLAB. Turn in your hand sketches and the MATLAB results on the same scales.

(a) $L(s) = \dfrac{1}{s^2(s+8)}$

(b) $L(s) = \dfrac{1}{s^3(s+8)}$

(c) $L(s) = \dfrac{1}{s^4(s+8)}$

(d) $L(s) = \dfrac{(s+3)}{s^2(s+8)}$
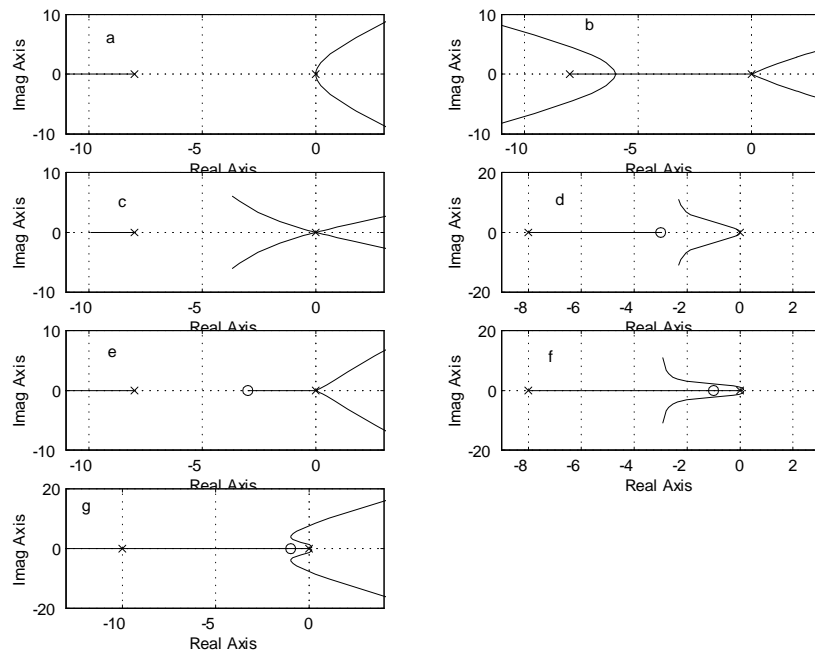
(e) $L(s) = \dfrac{(s+3)}{s^3(s+4)}$

(f) $L(s) = \dfrac{(s+1)^2}{s^3(s+4)}$

(g) $L(s) = \dfrac{(s+1)^2}{s^3(s+10)^2}$

Solution:

All the root locus plots are displayed at the end of the solution set for this problem.

(a) $\alpha = -2.67$; $\phi_i = \pm 60$; ± 180; $w_0 -> none$

(b) $\alpha = -2$; $\phi_i = \pm 45$; ± 135; $w_0 -> none$

(c) $\alpha = -1.6$; $\phi_i = \pm 36$; ± 108; $w_0 -> none$

(d) $\alpha = -2.5$; $\phi_i = \pm 90$; $w_0 -> none$

(e) $\alpha = -0.33$; $\phi_i = \pm 60$; ± 180; $w_0 -> none$

(f) $\alpha = -3$; $\phi_i = \pm 90$; $w_0 = \pm 1.414$

(g) $\alpha = -6$; $\phi_i = \pm 60$; 180; $w_0 = \pm 1.31; \pm 7.63$

Solution for Problem 5.6

7. *Mixed real and complex poles* Sketch the root locus with respect to K for the equation $1 + KL(s) = 0$ and the following choices for $L(s)$. Be sure to give the asymptotes, arrival and departure angles at any complex zero or pole, and the frequency of any imaginary-axis crossing. After completing each hand sketch verify your results using MATLAB. Turn in your hand sketches and the MATLAB results on the same scales.

(a) $L(s) = \dfrac{(s+2)}{s(s+10)(s^2+2s+2)}$

(b) $L(s) = \dfrac{(s+2)}{s^2(s+10)(s^2+6s+25)}$

(c) $L(s) = \dfrac{(s+2)^2}{s^2(s+10)(s^2+6s+25)}$

(d) $L(s) = \dfrac{(s+2)(s^2+4s+68)}{s^2(s+10)(s^2+4s+85)}$

(e) $L(s) = \dfrac{[(s+1)^2+1]}{s^2(s+2)(s+3)}$

Solution:

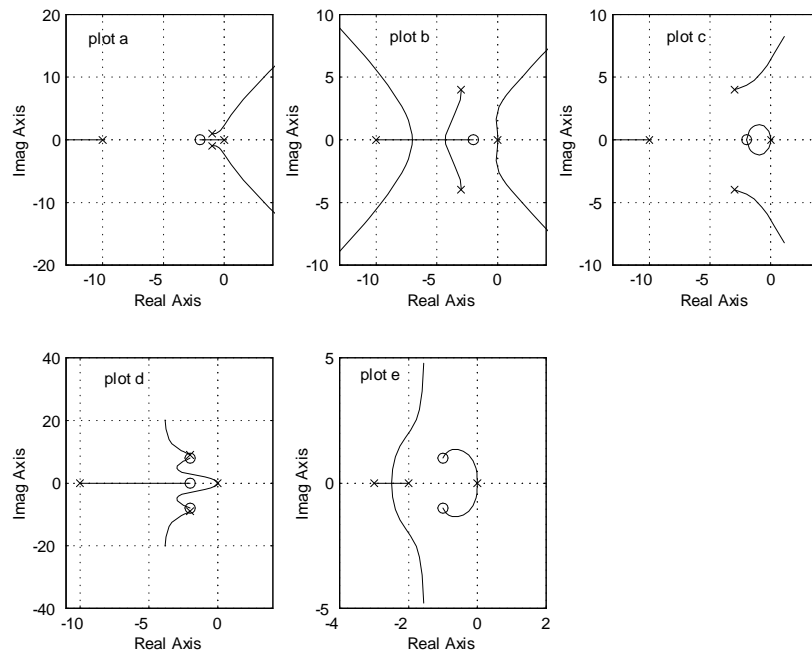All the plots are attached at the end of the solution set.

(a) $\alpha = -3.33;\ \phi_i = \pm 60;\ \pm 180;\ w_0 = \pm 2.32; \qquad \theta_d = \pm 6.34$

(b) $\alpha = -3.5;\ \phi_i = \pm 45;\ \pm 135;\ w_0 -> none; \qquad \theta_d = \pm 103.5$

(c) $\alpha = -4;\ \phi_i = \pm 60;\ \pm 180;\ w_0 = \pm 6.41; \qquad \theta_d = \pm 14.6$

(d) $\alpha = -4;\ \phi_i = \pm 90;\ w_0 -> none; \qquad \theta_d = \pm 106; \qquad \theta_a = \pm 253.4$

(e) $\alpha = -1.5;\ \phi_i = \pm 90;\ w_0 -> none; \qquad \theta_a = \pm 71.6$

Solution for Problem 5.7

8. *Right half plane poles and zeros* Sketch the root locus with respect to K for the equation $1 + KL(s) = 0$ and the following choices for $L(s)$. Be sure to give the asymptotes, arrival and departure angles at any complex zero or pole, and the frequency of any imaginary-axis crossing. After completing each hand sketch verify your results using MATLAB. Turn in your hand sketches and the MATLAB results on the same scales.

(a) $L(s) = \dfrac{s+2}{s+10} \dfrac{1}{s^2 - 1}$; The model for a case of magnetic levitation with lead compensation.

(b) $L(s) = \dfrac{s+2}{s(s+10)} \dfrac{1}{(s^2 - 1)}$; The magnetic levitation system with integral control and lead compensation.

(c) $L(s) = \dfrac{s-1}{s^2}$

(d) $L(s) = \dfrac{s^2 + 2s + 1}{s(s+20)^2(s^2 - 2s + 2)}$. What is the largest value that can be obtained for the damping ratio of the stable complex roots on this locus?

(e) $L(s) = \dfrac{(s+2)}{s(s-1)(s+6)^2}$,

(f) $L(s) = \dfrac{1}{(s-1)[(s+2)^2 + 3]}$

Solution:
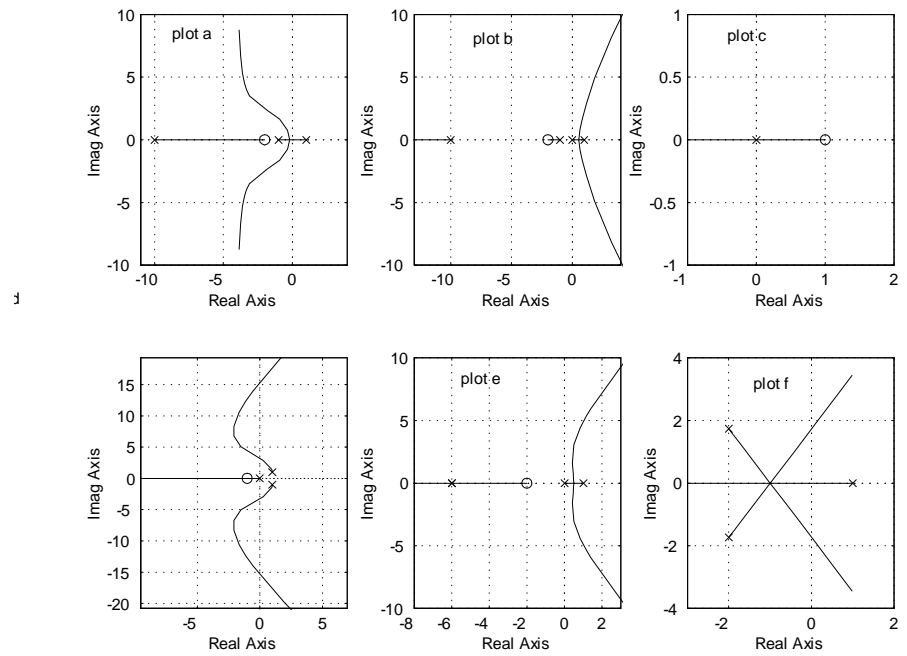
(a) $\alpha = -4$; $\phi_i = \pm 90$; $w_0 -> none$

(b) $\alpha = -4$; $\phi_i = \pm 60; 180$; $w_0 -> none$

(c) $\alpha = -1$; $\phi_i = \pm 180$; $w_0 -> none$

(d) $\alpha = -12$; $\phi_i = \pm 60; 180$; $w_0 = \pm 3.24; \pm 15.37$; $\qquad \theta_d = \pm 92.4$

(e) $\alpha = -3$; $\phi_i = \pm 60; 180$; $w_0 -> none$

(f) $\alpha = -1$; $\phi_i = \pm 60; 180$; $w_0 = \pm 1.732$; $\qquad \theta_d = \pm 40.9$

Solution for Problem 5.8

9. Put the characteristic equation of the system shown in Fig. 5.52 in root locus form with respect to the parameter α and identify the corresponding $L(s)$, $a(s)$, and $b(s)$. Sketch the root locus with respect to the parameter α, estimate the closed-loop pole locations and sketch the corresponding step responses when $\alpha = 0$, 0.5, and 2. Use MATLAB to check the accuracy of your approximate step responses.
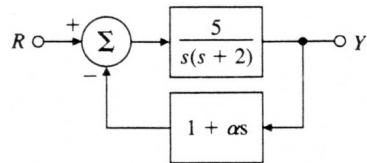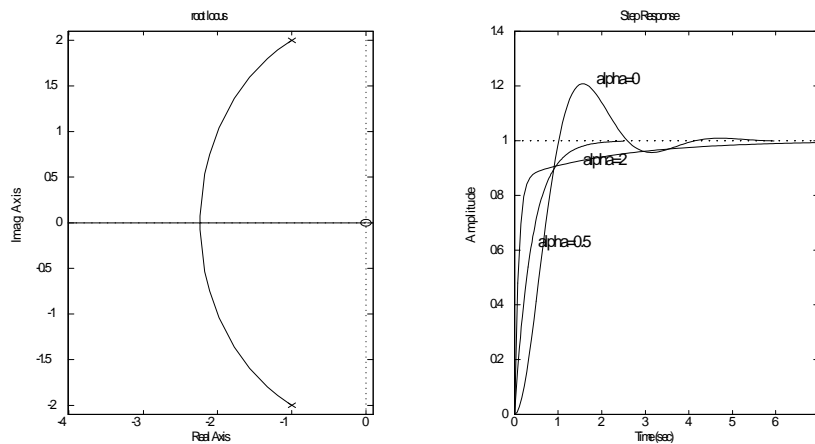

Figure 5.52: Control system for problem 9

Solution:

The characteristic equation is $s^2 + 2s + 5 + 5\alpha s = 0$ and $L(s) = \dfrac{s}{s^2 + 2s + 5}$. the root locus and step responses are plotted below.

10. Use the MATLAB function rltool to study the behavior of the root locus
    of $1 + KL(s)$ for

$$L(s) = \frac{(s+a)}{s(s+1)(s^2 + 8s + 52)}$$

as the parameter a is varied from 0 to 10, paying particular attention to
the region between 2.5 and 3.5.   Verify that a multiple root occurs at a
complex value of s for some value of a in this range.

Solution:

For small values of α, the locus branch from $0, -1$ makes a circular path
around the zero and the branches from the complex roots curve off toward
the asymptotes.   For large values of α the branches from the complex
roots break into the real axis and those from $0, -1$ curve off toward the
asymptotes. At about $\alpha = 3.11$ these loci touch corresponding to complex
multiple roots.

11. Use the Routh criterion to find the range of the gain K for which the systems in Fig. 5.53 are stable and use the root locus to confirm your calculations.
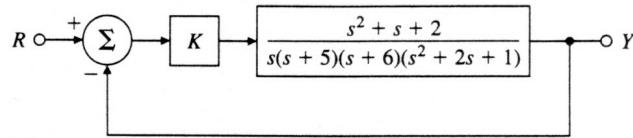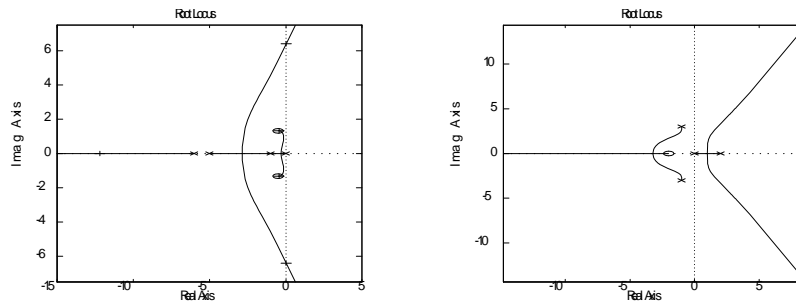

(a)
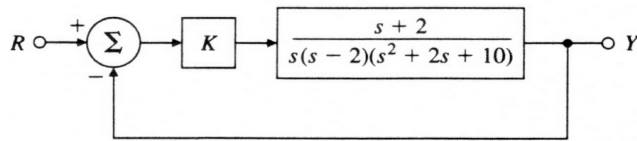
(b)

Figure 5.53: Feedback systems for problem 11

Solution:

(a) The system is stable for $0 \leq K \leq 478.226$ The root locus of the system and the location of the roots at the crossover points are shown in the plots

(b) There is a pole in the right hand plane thus the system is unstable for all values of K as shown in the last plot.


Solution for Problem 11

12. Sketch the root locus for the characteristic equation of the system   for which

$$L(s) = \frac{(s+2)}{s(s+1)(s+5)},$$

and determine the value of the root-locus gain for which the complex conjugate poles have a damping ratio of 0.5.

Solution:

Plot the system on Matlab using rlocus(sys), and use [K]= rlocfind(sys) to pick the gain where the damping ratio = 0.5.   Find that K = 14 (approximately).
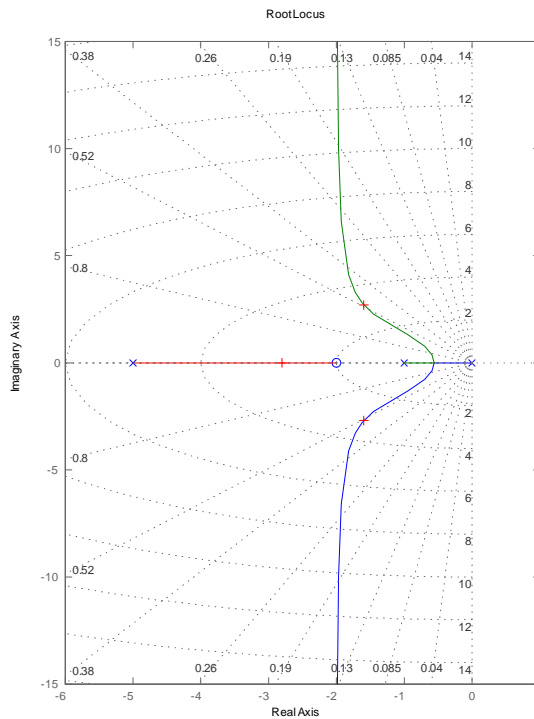
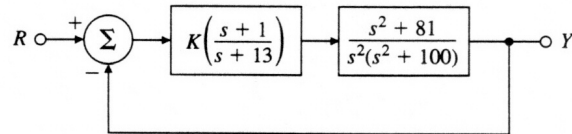Root locus with 0.5 damping marked

13. For the system in Fig. 5.54:

$$R \xrightarrow{+} \Sigma \rightarrow K\left(\frac{s+1}{s+13}\right) \rightarrow \frac{s^2+81}{s^2(s^2+100)} \rightarrow Y$$

Figure 5.54: Feedback system for problem 13

(a) Find the locus of closed-loop roots with respect to K.

(b) Is there a value of K that will cause all roots to have a damping ratio greater than 0.5?

(c) Find the values of K that yield closed-loop poles with the damping ratio $\zeta = 0.707$.

(d) Use MATLAB to plot the response of the resulting design to a reference step.

Solution:

(a) The locus is plotted below

(b) There is a K which will make the 'dominant' poles have damping 0.5 but none that will make the poles from the resonance have that much damping.

(c) Using rlocfind, the gain is about 35.

(d) The step response shows the basic form of a well damped response with the vibration of the response element added.
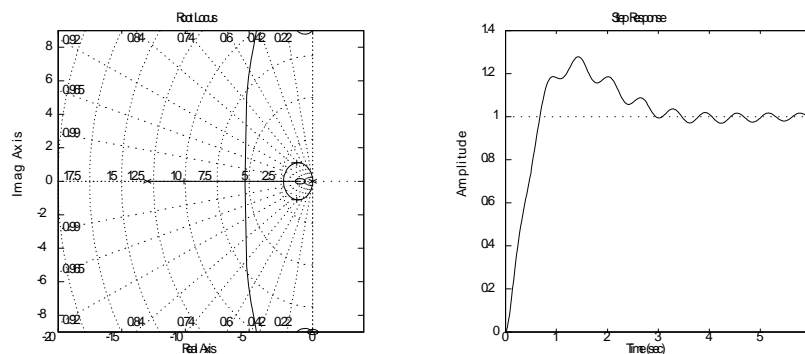
Root locus and step response for Problem 5.13

14. For the feedback system shown in Fig. 5.55, find the value of the gain K that results in dominant closed-loop poles with a damping ratio $\zeta = 0.5$.
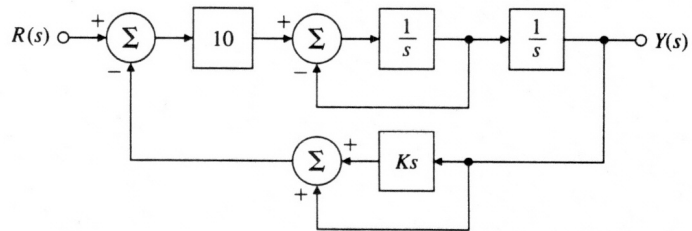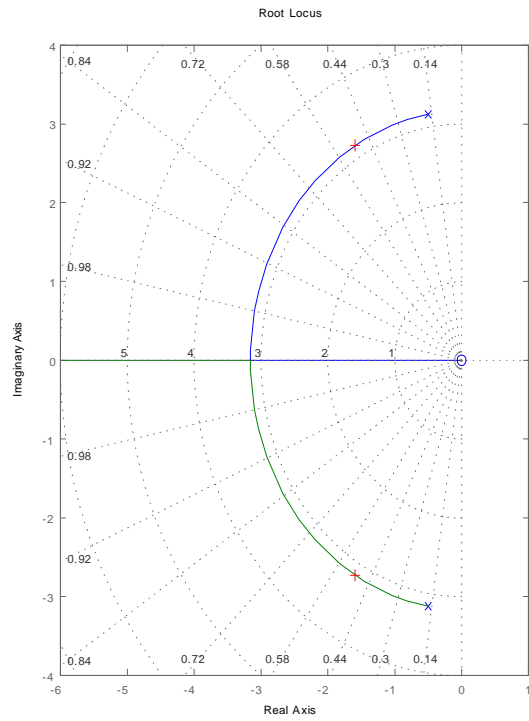

Figure 5.55: Feedback system for Problem 14

Solution:

Use block diagram reduction to find the characteristic equation of the closed loop system, then divide that up into terms with and without K to find the root locus form, where $L(s) = \dfrac{10s}{s^2 + s + 10}$. Plugging into Matlab and using rlocfind produces the required gain to be $K = 0.22$. The locus is

Root locus with 0.5 damping marked

Problems and solutions for Section 5.3

15. A simplified model of the longitudinal motion of a certain helicopter near hover has the transfer function

$$G(s) = \frac{9.8(s^2 - 0.5s + 6.3)}{(s + 0.66)(s^2 - 0.24s + 0.15)}.$$

and the characteristic equation $1 + D(s)G(s) = 0$. Let $D(s) = k_p$ at first.

   (a) Compute the departure and arrival angles at the complex poles and zeros.

   (b) Sketch the root locus for this system for parameter $K = 9.8k_p$. Use axes -4 $\leq x \leq$ 4. $-3 \leq y \leq 3$;

   (c) Verify your answer using MATLAB. Use the command axis([-4 4 -3 3]) to get the right scales.

   (d) Suggest a practical (at least as many poles as zeros) alternative compensation $D(s)$ which will at least result in a stable system.

   Solution:

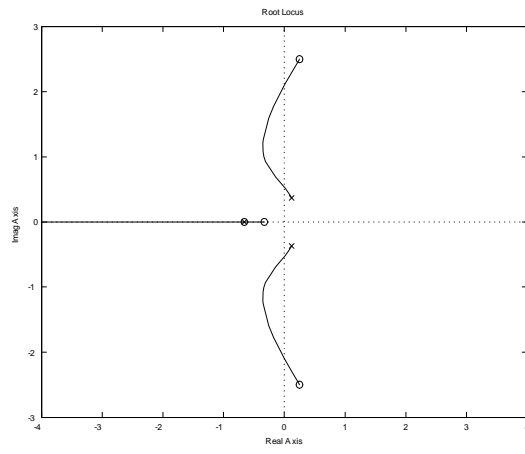   (a) $\alpha = .92$; $\phi = 180$; $\varphi = 63.83$; $\psi = -26.11$
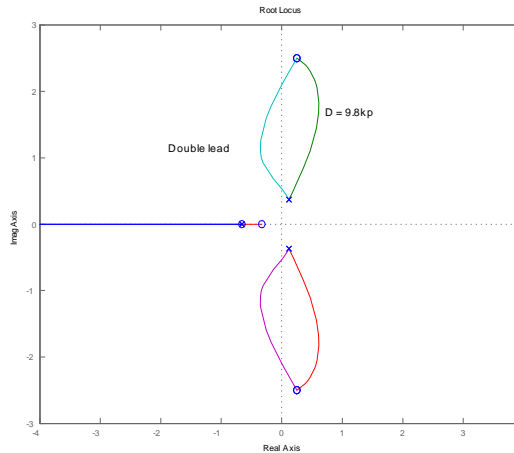
   (b)

   Problem 5.15(b)

   (c) For this problem a double lead is needed to bring the roots into the left half-plane. The plot shows the rootlocus for control for. Let
$$D = \frac{(s + .66)(s + .33)}{(s + 5)^2} \quad .$$
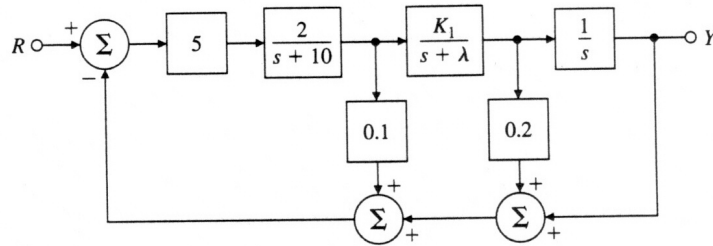
(d)

Problem 5.15(d)

Figure 5.56: Control system for problem 5.16

16. For the system given in Fig. 5.56,

    (a) plot the root locus of the characteristic equation as the parameter K_1 is varied from 0 to ∞ with $\lambda = 2$. Give the corresponding $L(s)$, $a(s)$, and $b(s)$.

    (b) Repeat part (a) with $\lambda = 5$. Is there anything special about this value?

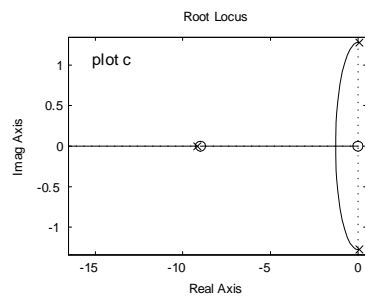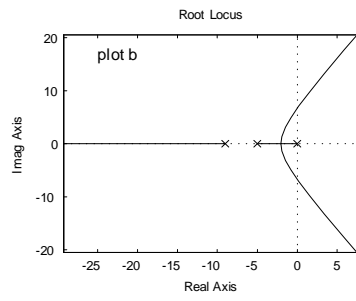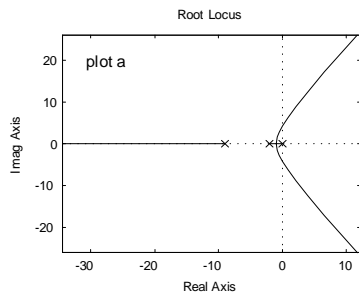   (c) Repeat part (a) for fixed $K_1 = 2$ with the parameter $K = \lambda$ varying from 0 to ∞.

        Solution:

        The root locus for each part is attached at the end.

    (a) $L(s) = \dfrac{0.75}{S(0.1S^2 + 1.1S + 1.8)} = \dfrac{a(s)}{b(s)}$

    (b) $L(s) = \dfrac{0.75}{S(0.1S^2 + 1.4S + 4.5)} = \dfrac{a(s)}{b(s)}$

    (c) $L(s) = \dfrac{S(0.1S + 0.9)}{0.1S\textasciicircum 3 + 0.9S + 1.5} = \dfrac{a(s)}{b(s)}$
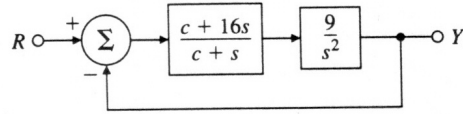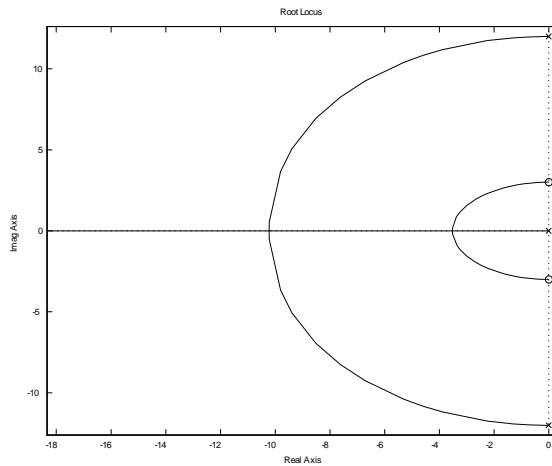
Solution for problem 5.16

Figure 5.57: Control system for problem 17

17. For the system shown in Fig. 5.57, determine the characteristic equation and sketch the root locus of it with respect to positive values of the parameter c. Give $L(s)$, $a(s)$, and $b(s)$ and be sure to show with arrows the direction in which c increases on the locus.

(a) Solution:

$$L(s) = \frac{s^2 + 9}{s^3 + 144s} = \frac{a(s)}{b(s)}$$

Solution for problem 5.17

18. Suppose you are given a system with the transfer function

$$L(s) = \frac{(s+z)}{(s+p)^2},$$

where z and p are real and $z > p$. Show that the root-locus for $1+KL(s) = 0$ with respect to K is a circle centered at z with radius given by

$$r = (z - p)$$

Hint. Assume $s + z = re^{j\phi}$ and show that $L(s)$ is real and negative for real ϕ under this assumption.

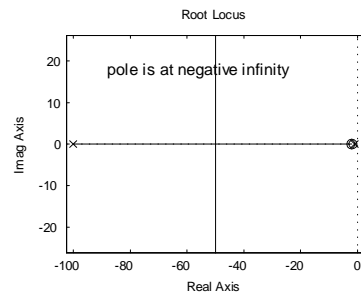**Solution:**

$s + z = (z - p)e^{j\phi}$

$$G = \frac{(z-p)e^{j\phi}}{((z-p)e^{j\phi} + p - z)^2} = \frac{(z-p)e^{j\phi}}{(z-p)^2(e^{j\phi}-1)^2} = \frac{1}{(z-p)(-4)(\frac{e^{j\phi/2}-e^{-j\phi/2}}{2j})^2}$$

$$= \frac{1}{-4(z-p)} \frac{1}{(\sin(\phi/2))^2}$$ Because $z > p$, this function is real and negative for real ϕ and therefore these points are on the locus.

19. The loop transmission of a system has two poles at $s = -1$ and a zero at $s = -2$. There is a third real-axis pole p located somewhere to the *left* of the zero. Several different root loci are possible, depending on the exact location of the third pole. The extreme cases occur when the pole is located at infinity or when it is located at $s = -2$. Give values for p and sketch the three distinct types of loci.
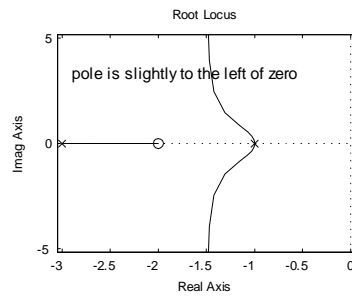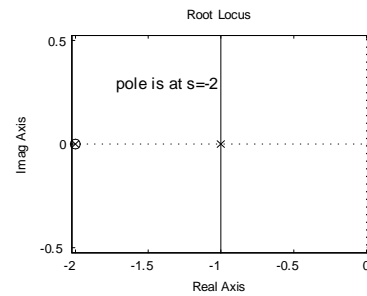
Solution for problem 5.19

20. For the feedback configuration of Fig. 5.58, use asymptotes, center of asymptotes, angles of departure and arrival, and the Routh array to sketch root loci for the characteristic equations of the following feedback control systems versus the parameter K. Use MATLAB to verify your results.

(a) $G(s) = \dfrac{1}{s(s+1+3j)(s+1-3j)}$, $\qquad\qquad\qquad H(s) = \dfrac{s+2}{s+8}$

(b) $G(s) = \dfrac{1}{s^2}$, $\qquad\qquad\qquad\qquad\qquad\quad H(s) = \dfrac{s+1}{s+3}$

(c) $G(s) = \dfrac{(s+5)}{(s+1)}$, $\qquad\qquad\qquad\qquad\quad H(s) = \dfrac{s+7}{s+3}$

(d) $G(s) = \dfrac{(s+3+4j)(s+3-4j)}{s(s+1+2j)(s+1-2j)}$, $\qquad H(s) = 1+3s$
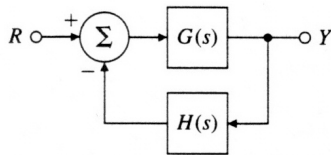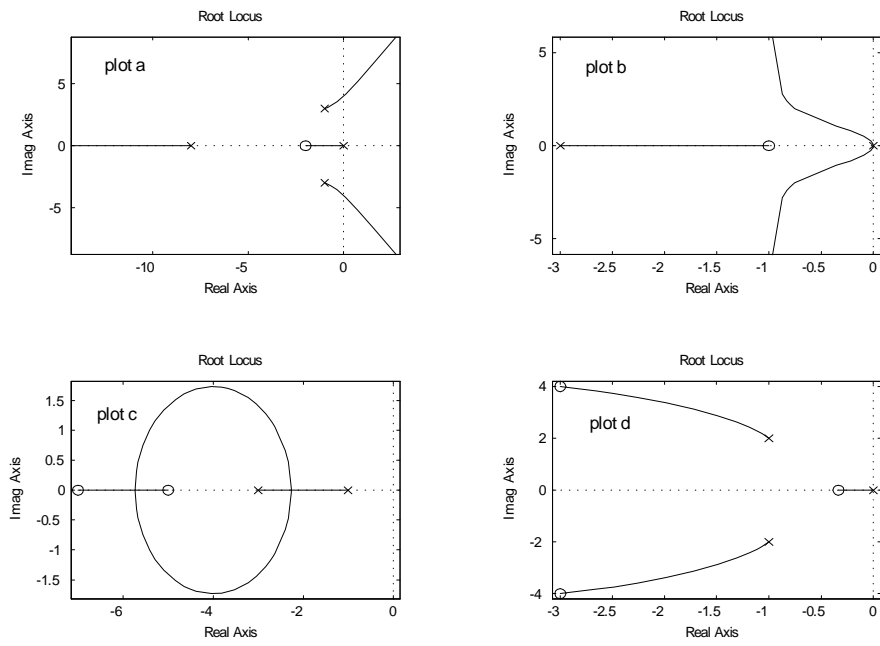

Figure 5.58: Feedback system for problem 20

Solution:

Solution for problem 5.20

21. Consider the system in Fig. 5.59.
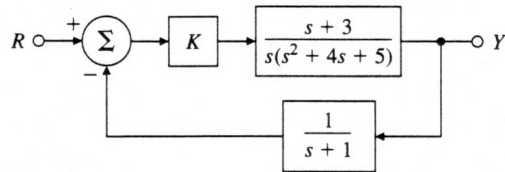
Figure 5.59: Feedback system for problem 5.21

(a) Using Routh's stability criterion, determine all values of K for which the system is stable.

(b) Use Matlab to find the root locus versus K. Find the values for K at imaginary-axis crossings.

Solution:

(a) a. $0 \leq K \leq 40$

(b) $\theta_d = \pm 161.6°$ $\theta_a = 0°$

At imaginary axis crossing $s = \pm j1.8186$ $k = 6.2758$

Root locus is attached for reference.
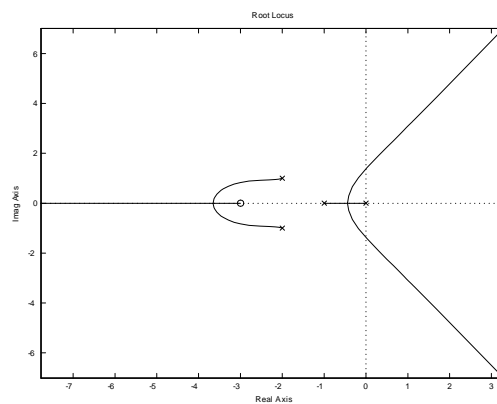
Root locus for problem 21

Problems and solutions for Section 5.4

22. Let

$$G(s) = \frac{1}{(s+2)(s+3)} \quad \text{and} \quad D(s) = K\frac{s+a}{s+b}.$$

Using root-locus techniques, find values for the parameters a, b, and K of the compensation $D(s)$ that will produce closed-loop poles at $s = -1 \pm j$ for the system shown in Fig. 5.60.
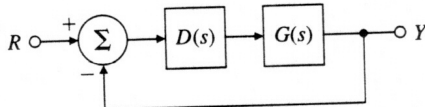

Figure 5.60: Unity feedback system for Problems 5.22 to 5.28 and 5.33

Solution:

Since the desired poles are slower than he plant, we will use PI control. The solution is to cancel the pole at -3 with the zero and set the gain to $K = 2$. Thus, $p = 0$, $z = -3$, $K = 2$.

23. Suppose that in Fig. 5.60,
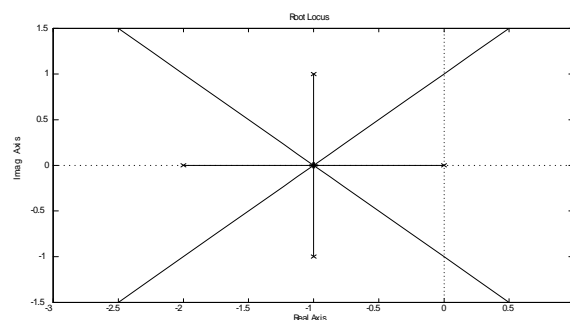
$$G(s) = \frac{1}{s(s^2 + 2s + 2)} \quad \text{and} \quad D(s) = \frac{K}{s + 2}.$$

Sketch the root-locus with respect to K of the characteristic equation for the closed-loop system, paying particular attention to points that generate multiple roots if $KL(s) = D(s)G(s)$.

Solution:

The locus is plotted below. The roots all come together at $s = -1$ at $K = 1$.

Root locus for Problem 23

24. Suppose the unity feedback system of Fig. 5.60 has an open-loop plant given by $G(s) = 1/s^2$. Design a lead compensation $D(s) = K\dfrac{s+z}{s+p}$ to be added in series with the plant so that the dominant poles of the closed-loop system are located at $s = -2 \pm 2j$.

Solution:

Setting the pole of the lead to be at $p = -20$, the zero is at $z = -1.78$ with a gain of $K = 72$. The locus is plotted below.
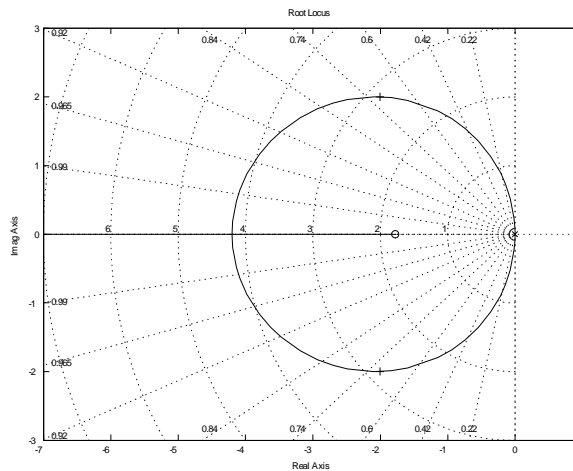
Root locus for Problem 24

25. Assume that the unity feedback system of Fig. 5.60 has the open-loop plant

$$G(s) = \frac{1}{s(s+3)(s+6)}.$$

Design a lag compensation to meet the following specifications:

- The step response settling time is to be less than 5 sec.

- The step response overshoot is to be less than 17%.

- The steady-state error to a unit ramp input must not exceed 10%.

   Solution:

   The overshoot specification requires that damping be 0.5 and the settling time requires that $\omega_n > 1.8$. From the root locus plotted below, these can be met at $K = 28$ where the $\omega_n = 2$. With this gain, the $K_v = 28/18 = 1.56$. To get a $K_v = 10$, we need a lag gain of about 6.5. Selecting the lag zero to be at 0.1 requires the pole to be at $0.1/6.5 = 0.015$. To meet the overshoot specifications, it is necessary to select a smaller K and set $p = 0.01$. Other choices are of course possible. The step response of this design is plotted below.
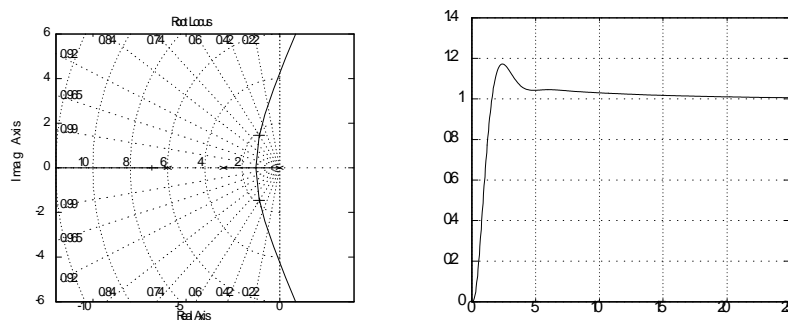
Root locus and step response for Problem 5.25

26. A numerically controlled machine tool positioning servomechanism has a normalized and scaled transfer function given by

$$G(s) = \frac{1}{s(s+1)}.$$

Performance specifications of the system in the unity feedback configuration of Fig. 5.60 are satisfied if the closed-loop poles are located at $s = -1 \pm j\sqrt{3}$.

(a) Show that this specification cannot be achieved by choosing proportional control alone, $D(s) = k_p$.

(b) Design a lead compensator $D(s) = K\dfrac{s+z}{s+p}$ that will meet the specification.

Solution:

(a) With proportional control, the poles have real part at $s = -.5$.

(b) To design a lead, we select the pole to be at $p = -10$ and find the zero and gain to be $z = -3$, $k = 12$.

27. A servomechanism position control has the plant transfer function

$$G(s) = \frac{10}{s(s+1)(s+10)}.$$

You are to design a series compensation transfer function $D(s)$ in the unity feedback configuration to meet the following closed-loop specifications:

- The response to a reference step input is to have no more than 16% overshoot.

- The response to a reference step input is to have a rise time of no more than 0.4 sec.

- The steady-state error to a unit ramp at the reference input must be less than 0.02

(a) Design a lead compensation that will cause the system to meet the dynamic response specifications.

(b) If $D(s)$ is proportional control, $D(s) = k_p$, what is the velocity constant K_v?

(c) Design a lag compensation to be used in series with the lead you have designed to cause the system to meet the steady-state error specification.

(d) Give the MATLAB plot of the root locus of your final design.

(e) Give the MATLAB response of your final design to a reference step .

Solution:

(a) Setting the lead pole at $p = -60$ and the zero at $z = -1$, the dynamic specifications are met with a gain of 245 resulting in a $K_v = 4$.

(b) Proportional control will not meet the dynamic spec. The K_v of the lead is given above.

(c) To meet the steady-state requirement, we need a new $K_v = 50$, which is an increase of 12.5. If we set the lag zero at $z = -.4$, the pole needs to be at $p = -0.032$.

(d) The root locus is plotted below.

(e) The step response is plotted below.
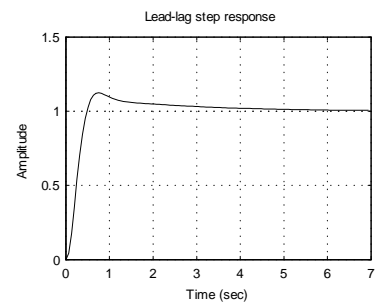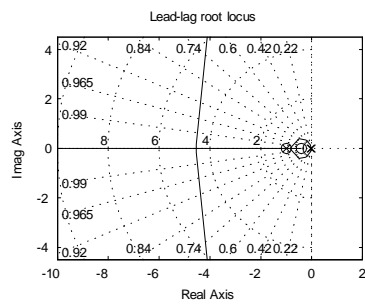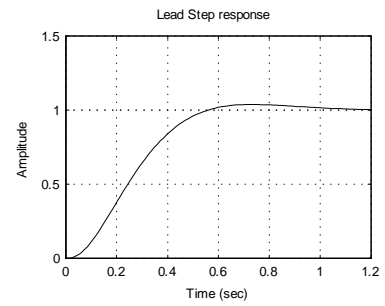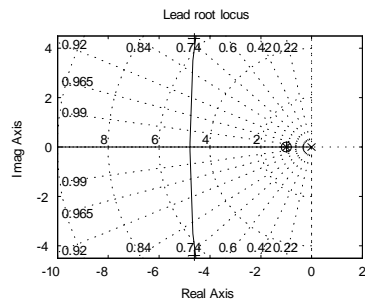
Solution to Problem 27

28. Assume the closed-loop system of Fig. 5.60 has a feed forward transfer function $G(s)$ given by

$$G(s) = \frac{1}{s(s+2)}.$$

Design a lag compensation so that the dominant poles of the closed-loop system are located at $s = -1 \pm j$ and the steady-state error to a unit ramp input is less than 0.2.

Solution:

The poles can be put in the desired location with proportional control alone, with a gain of $k_p = 2$ resulting in a $K_v = 1$. To get a $K_v = 5$, we add a compensation with zero at 0.1 and a pole at 0.02. $D(s) = 2\dfrac{s + 0.1}{s + 0.02}$.

29. An elementary magnetic suspension scheme is depicted in Fig. 5.61. For small motions near the reference position, the voltage e on the photo detector is related to the ball displacement x (in meters) by $e = 100x$. The upward force (in newtons) on the ball caused by the current i (in amperes) may be approximated by $f = 0.5i + 20x$. The mass of the ball is 20 g, and the gravitational force is 9.8 N/kg. The power amplifier is a voltage-to-current device with an output (in amperes) of $i = u + V_0$.
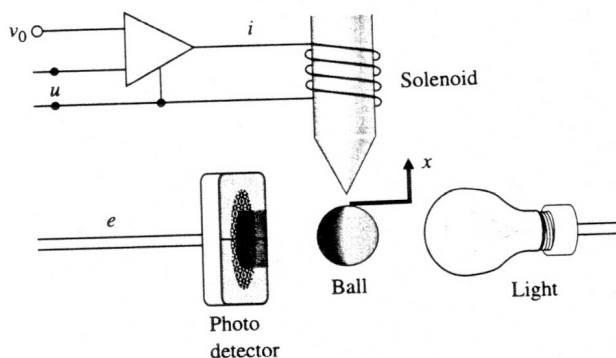

Figure 5.61: Elementary magnetic suspension

(a) Write the equations of motion for this setup.

(b) Give the value of the bias V_0 that results in the ball being in equilibrium at $x = 0$.

(c) What is the transfer function from u to e?

(d) Suppose the control input u is given by $u = -Ke$. Sketch the root locus of the closed-loop system as a function of K.

(e) Assume that a lead compensation is available in the form $\dfrac{U}{E} = D(s) = K\dfrac{s+z}{s+p}$. Give values of K, z, and p that yields improved performance over the one proposed in part (d).

Solution:

(a) $m\ddot{x} = 20x + 0.5i - mg$. Substituting numbers, $0.02\ddot{x} = 20x + 0.5(u + V_o) - 0.196$.

(b) To have the bias cancel gravity, the last two terms must add to zero. Thus $V_o = 0.392$.

(c) Taking transforms of the equation and substituting $e = 100x$,

$$\frac{E}{U} = \frac{2500}{s^2 - 1000}$$

(d) The locus starts at the two poles symmetric to the imaginary axis, meet at the origin and cover the imaginary axis. The locus is plotted below.

(e) The lead can be used to cancel the left-hand-plane zero and the pole at m$-$150 which will bring the locus into the left-hand plane where K can be selected to give a damping of, for example 0.7. See the plot below.
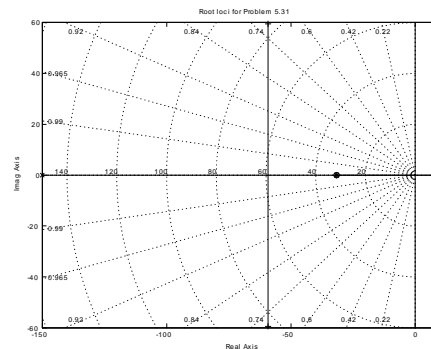
Root loci for Problem 29

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30. A certain plant with the non minimum phase transfer function

$$G(s) = \frac{4 - 2s}{s^2 + s + 9},$$

is in a unity positive feedback system with the controller transfer function $D(s)$.

(a) Use MATLAB to determine a (negative) value for $D(s) = K$ so that the closed-loop system with negative feedback has a damping ratio $\zeta = 0.707$.

(b) Use MATLAB to plot the system's response to a reference step.

   Solution:

(a) With all the negatives, the problem statement might be confusing. With the $G(s)$ as given, MATLAB needs to plot the negative locus, which is the regular positive locus for $-G$. The locus is plotted below. The value of gain for closed loop roots at damping of 0.7 is $k = -1.04$

(b) The final value of the step response plotted below is -0.887. To get a positive output we would use a positive gain in positive feedback.
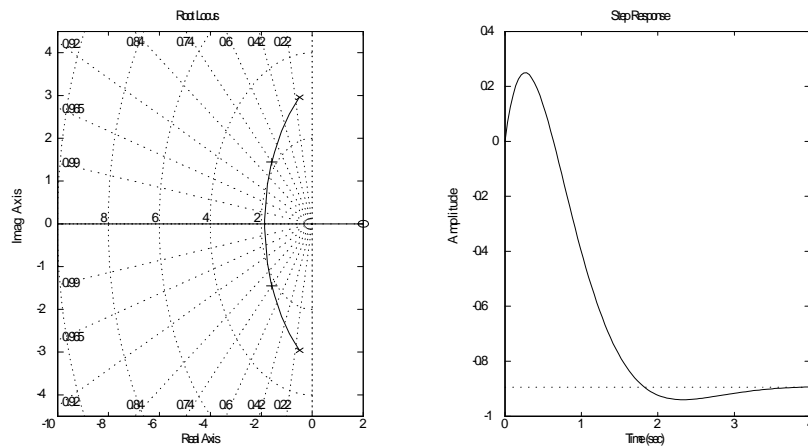
Solutions for Problem 30

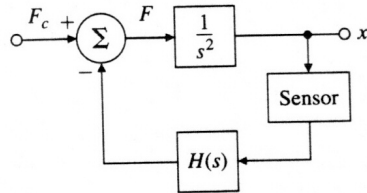31. Consider the rocket-positioning system shown in Fig. 5.62.

Figure 5.62: Block diagram for rocket-positioning control system

(a) Show that if the sensor that measures x has a unity transfer function, the lead compensator

$$H(s) = K\frac{s+2}{s+4}$$

stabilizes the system.

(b) Assume that the sensor transfer function is modeled by a single pole with a 0.1 sec time constant and unity DC gain. Using the root-locus procedure, find a value for the gain K that will provide the maximum damping ratio.

Solution:

(a) The root locus is plotted below and lies entirely in the left-half plane. However the maximum damping is 0.2.

(b) At maximum damping, the gain is $K = 6.25$ but the damping of the complex poles is only 0.073. A practical design would require much more lead.
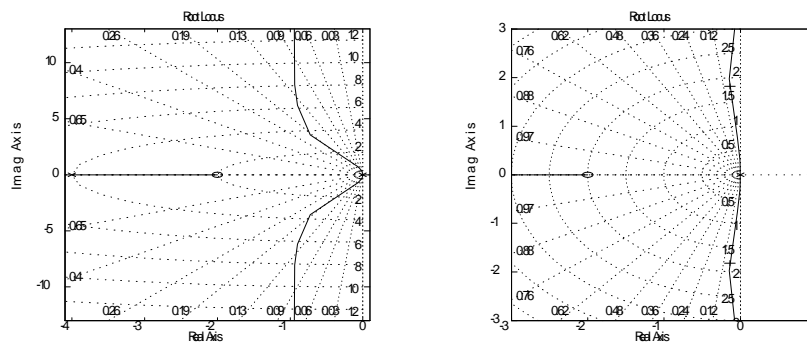
Loci for problem 31
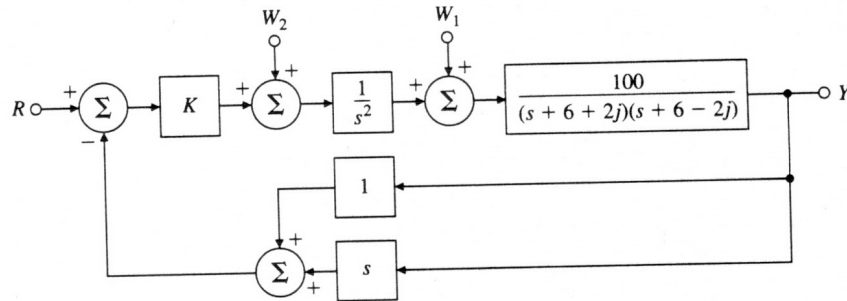
32. For the system in Fig. 5.63:

Figure 5.63: Control system for Problem 32

(a) Sketch the locus of closed-loop roots with respect to K.

(b) Find the maximum value of K for which the system is stable. Assume $K = 2$ for the remaining parts of this problem.

(c) What is the steady-state error $(e = r - y)$ for a step change in r?

(d) What is the steady-state error in y for a constant disturbance w_1?

(e) What is the steady-state error in y for a constant disturbance w_2?

(f) If you wished to have more damping, what changes would you make to the system?

Solution:

(a) For the locus, $L(s) = \dfrac{100(s+1)}{s^2(s^2 + 12s + 40)}$. The locus is plotted below.
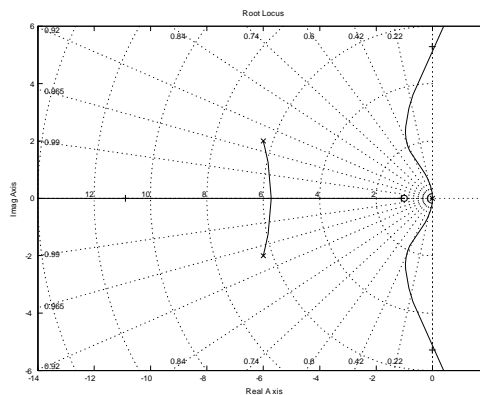
Locus for Problem 32

(b) The maximum value of K for stability is $K = 3.35$.

(c) The equivalent plant with unity feedback is $G' = \dfrac{200}{s^2(s^2+12+40)+200s}$. Thus the system is **type** 1 with $K_v = 1$. If the velocity feedback were zero, the system would be type 2 with $K_a = \dfrac{200}{40} = 5$.

(d) The transfer function $\dfrac{Y}{W_1} = \dfrac{100s^2}{s^2(s^2+12s+40)+200(s+1)}$. The system is thus **type 2** with $K_a = 100$.

(e) The transfer function $\dfrac{Y}{W_2} = \dfrac{100}{s^2(s^2+12s+40)+200(s+1)}$. The system here is **type 0** with $K_p = 1$.

(f) To get more damping in the closed-loop response, the controller needs to have a lead compensation.

33. Consider the plant transfer function

$$G(s) = \frac{bs+k}{s^2[mMs^2+(M+m)bs+(M+m)k]}$$

to be put in the unity feedback loop of Fig. 5.60. This is the transfer function relating the input force $u(t)$ and the position $y(t)$ of mass M in the non-collocated sensor and actuator problem. In this problem we will use root-locus techniques to design a controller $D(s)$ so that the closed-loop step response has a rise time of less than 0.1 sec and an overshoot of less than 10%. You may use MATLAB for any of the following questions.

(a) Approximate $G(s)$ by assuming that $m \cong 0$, and let $M = 1$, $k = 1$, $b = 0.1$, and $D(s) = K$. Can K be chosen to satisfy the performance specifications? Why or why not?

(b) Repeat part (a) assuming $D(s) = K(s+z)$, and show that K and z can be chosen to meet the specifications.

(c) Repeat part (b) but with a practical controller given by the transfer function

$$D(s) = K\frac{p(s+z)}{s+p},$$

and pick p so that the values for K and z computed in part (b) remain more or less valid.

(d) Now suppose that the small mass m is not negligible, but is given by $m = M/10$. Check to see if the controller you designed in part (c) still meets the given specifications. If not, adjust the controller parameters so that the specifications are met.

Solution:

(a) The locus in this case is the imaginary axis and cannot meet the specs for any K.

(b) The specs require that $\zeta > 0.6$, $\omega_n > 18$. Select $z = 15$ for a start. The locus will be a circle with radius 15. Because of the zero, the overshoot will be increased and Figure 3.32 indicates that we'd better make the damping greater than 0.7. As a matter of fact, experimentation shows that we can lower the overshoot of less than 10% only by setting the zero at a low value and putting the poles on the real axis. The plot shows the result if $D = 25(s + 4)$.

(c) In this case, we take $D(s) = 20\dfrac{s+4}{.01s+1}$.

(d) With the resonance present, the only chance we have is to introduce a notch as well as a lead. The compensation resulting in the plots shown is $D(s) = 11\dfrac{s+4}{(.01s+1)}\dfrac{s^2/9.25 + s/9.25 + 1}{s^2/3600 + s/30 + 1}$. The design gain was obtained by a cycle of repeated loci, root location finding, and step responses. Refer to the file ch5p35.m for the design aid.
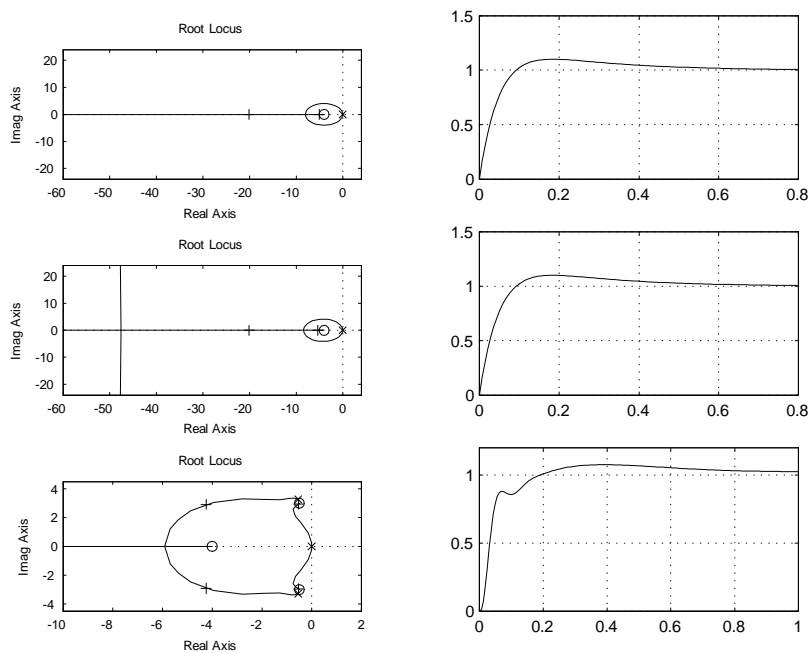
Root loci and step responses for Problem 33

34. Consider the type 1 system drawn in Fig. 5.64. We would like to design the compensation $D(s)$ to meet the following requirements: (1) The steady-state value of y due to a constant unit disturbance w should be less than $\frac{4}{5}$, and (2) the damping ratio $\zeta = 0.7$. Using root-locus techniques:
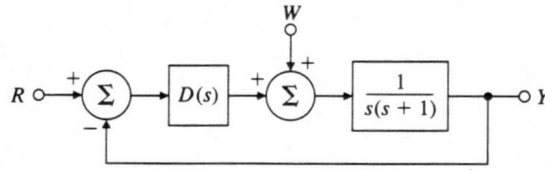
(a) Show that proportional control alone is not adequate.

Figure 5.64: Control system for problem 34

(b) Show that proportional-derivative control will work.

(c) Find values of the gains k_p and k_D for $D(s) = k_p + k_D\,s$ that meet the design specifications.

Solution:

(a) To meet the error requirements, the input to $D(s)$ is -0.8 and the output must be 1.0 to cacel the disturbance. Thus the controller dc gain must be at least 1.25. With proportional control and a closed loop damping of 0.70, the gain is 0.5 which is too low.

(b) With PD control, the characteristic equation is $s^2 + (1 + k_D)s + k_p$. Setting $k_p = 1.25$ and damping 0.7, we get $k_D = 0.57$. The root loci and disturbance step response are plotted below.

(c) The gains are $k_p = 1.25$, $k_D = 0.57$.
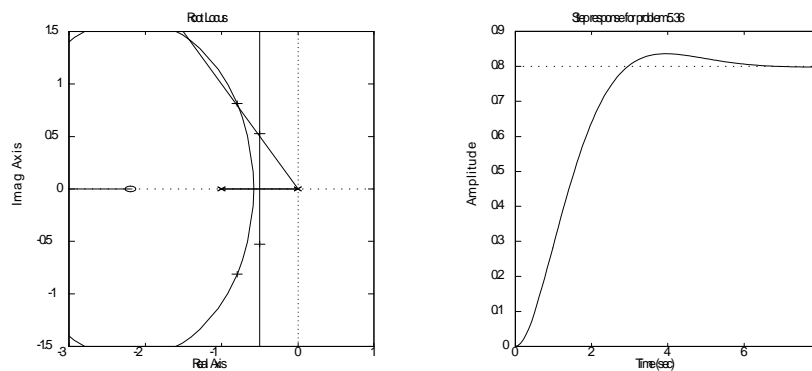
Solution for problem 34

Problems and solutions for Section 5.5

35. Consider the positioning servomechanism system shown in Fig. 5.65, where

$$e_i = K_{\text{pot}}\theta_i, \quad e_o = K_{\text{pot}}\theta_o, \quad K_{\text{pot}} = 10\text{V/rad},$$
$$T = \text{motor torque} = K_t i_a,$$
$$K_t = \text{torque constant} = 0.1 \text{ N}\cdot\text{m/A}, = K_e$$
$$R_a = \text{armature resistance} = 10\Omega,$$
$$\text{Gear ratio} = 1:1,$$
$$J_L + J_m = \text{total inertia} = 10^{-3} \text{ kg}\cdot\text{m}^2,$$
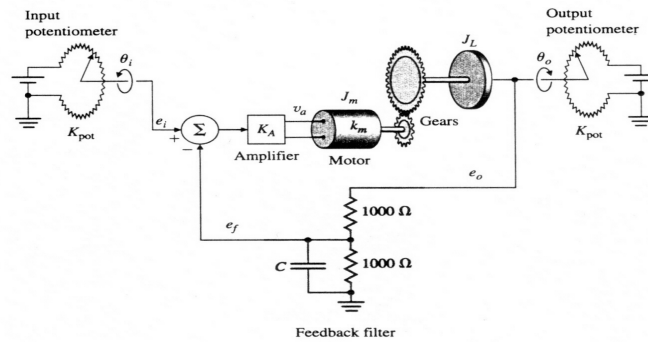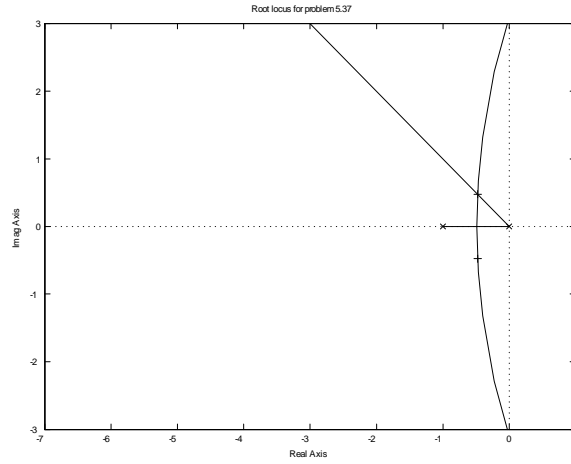$$C = 200\mu\text{F},$$
$$v_a = K_A(e_i - e_f).$$

Figure 5.65: Positioning servomechanism

(a) What is the range of the amplifier gain K_A for which the system is stable? Estimate the upper limit graphically using a root-locus plot.

(b) Choose a gain K_A that gives roots at $\zeta = 0.7$. Where are all three closed-loop root locations for this value of K_A?

Solution:

(a) $0 < K < 110$

Root locus for problem 35

$K = 10.$; poles are at $s = -10.05, -0.475 \pm j0.475$.

36. We wish to design a velocity control for a tape-drive servomechanism. The transfer function from current $I(s)$ to tape velocity $\Omega(s)$ (in millimeters per millisecond per ampere) is

$$\frac{\Omega(s)}{I(s)} = \frac{15(s^2 + 0.9s + 0.8)}{(s+1)(s^2 + 1.1s + 1)}.$$

We wish to design a type 1 feedback system so that the response to a reference step satisfies

$$t_r \leq 4\text{msec}, \quad t_s \leq 15\text{msec}, \quad M_p \leq 0.05$$

(a) Use the integral compensator k_I/s to achieve type 1 behavior, and sketch the root-locus with respect to k_I. Show on the same plot the region of acceptable pole locations corresponding to the specifications.

(b) Assume a proportional-integral compensator of the form $k_p(s+\alpha)/s$, and select the best possible values of k_p and α you can find. Sketch the root-locus plot of your design, giving values for k_p and α, and the velocity constant K_v your design achieves. On your plot, indicate the closed-loop poles with a dot •, and include the boundary of the region of acceptable root locations.

Solution:

(a) The root locus is plotted with the step response below in the first row.

(b) The zero was put at $s = -1.7$ and the locus and step response are plotted in the second row below
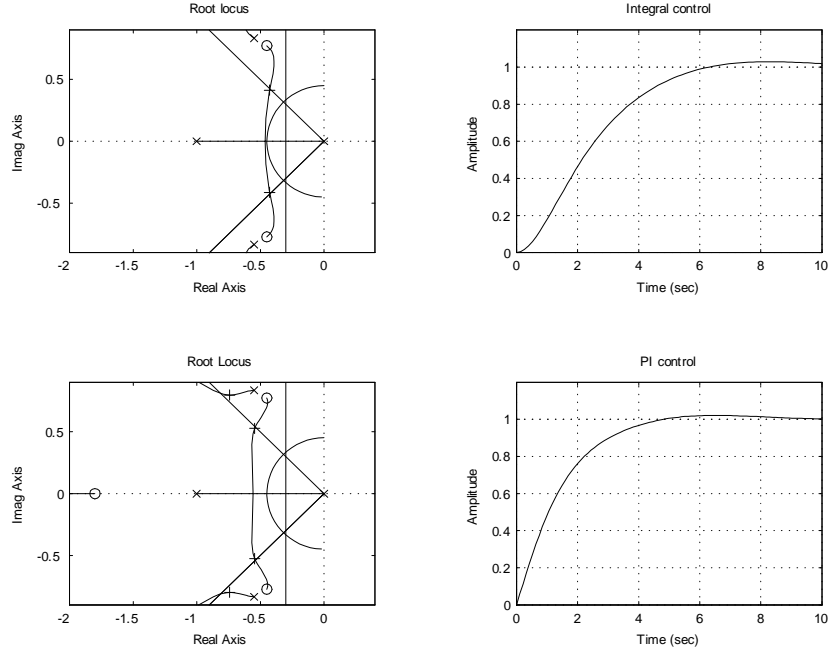
Solution for problem 36

37. The normalized, scaled equations of a cart as drawn in Fig. 5.66 of mass m_c holding an inverted uniform pendulum of mass m_p and length ℓ with no friction are

$$\ddot{\theta} - \theta = -v$$
$$\ddot{y} + \beta\theta = v$$
(5.1)

where $\beta = \dfrac{3m_p}{4(m_c + m_p)}$ is a mass ratio bounded by $0 < \beta < 0.75$. Time is measured in terms of $\tau = \omega_o t$ where $\omega_o^2 = \dfrac{3g(m_c + m_p)}{\ell(4m_c + m_p)}$. The cart motion, y, is measured in units of pendulum length as $y = \dfrac{3x}{4\ell}$ and the input is force normalized by the system weight, $v = \dfrac{u}{g(m_c + m_p)}$. These equations can be used to compute the transfer functions

$$\frac{\Theta}{V} = -\frac{1}{s^2 - 1}$$
(5.2)

$$\frac{Y}{V} = \frac{s^2 - 1 + \beta}{s^2(s^2 - 1)}$$
(5.3)

In this problem you are to design a control for this system by first closing a loop around the pendulum, Eq.(5.2) and then, with this loop closed,
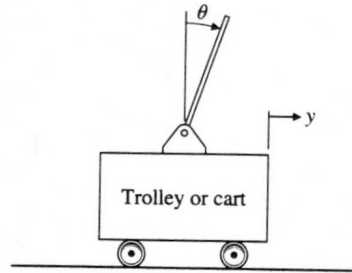
Figure 5.66: Figure of cart-pendulum for Problem 37

closing a second loop around the cart plus pendulum Eq.(5.3). For this problem, let the mass ratio be $m_c = 5m_p$.

(a) Draw a block diagram for the system with V input and both Y and Θ as outputs.

(b) Design a lead compensation $D_p(s) = K_p \dfrac{s+z}{s+p}$ for the Θ loop to cancel the pole at $s = -1$ and place the two remaining poles at $-4 \pm j4$. The new control is $U(s)$ where the force is $V(s) = U(s) + D(s)\Theta(s)$. Draw the root locus of the angle loop.

(c) Compute the transfer function of the new plant from U to Y with $D(s)$ in place.

(d) Design a controller $D_c(s)$ for the cart position with the pendulum loop closed. Draw the root locus with respect to the gain of $D_c(s)$

(e) Use MATLAB to plot the control, cart position, and pendulum position for a unit step change in cart position.
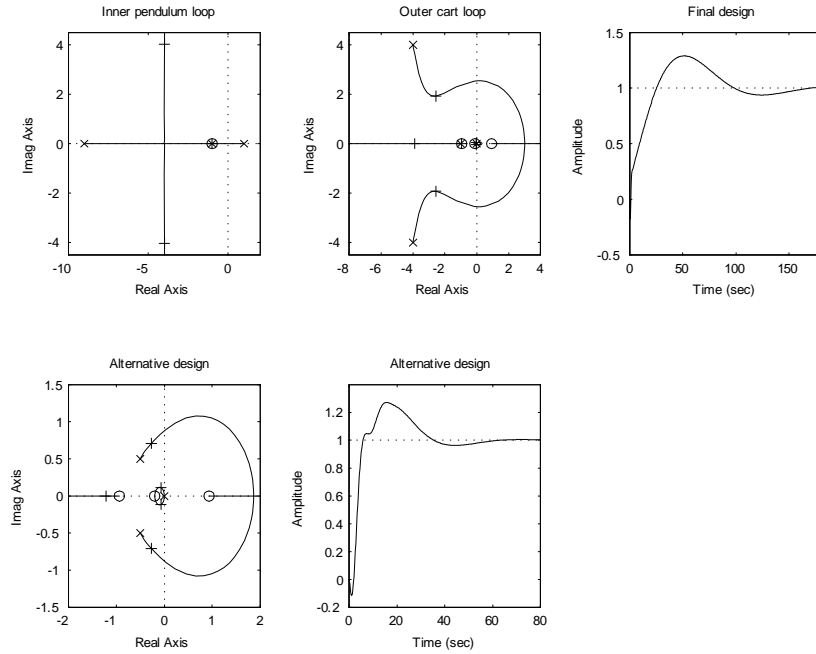
Solution:

(a)
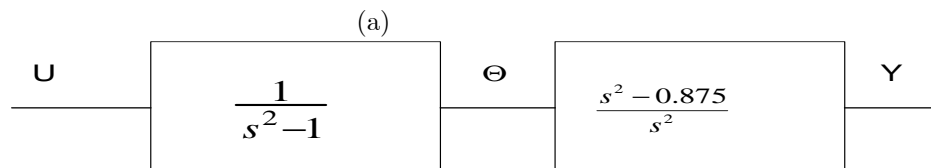
(b) $D_p(s) = 41\dfrac{s+1}{s+9}$ The root locus is shown below.

(c) $G_1 = \dfrac{-41}{s^2 + 8s + 32}\dfrac{s^2 - 0.875}{s^2}$

(d) $D_c = k_c\dfrac{s^2 + 0.2s + 0.01}{s^2 + 2s + 1}$.   The root locus is shown below.

(e) The step responses are shown below. The pendulum position control is rather fast for this problem. A more reasonable alternative choice would be to place the pendulum roots at $s = -0.5 \pm j0.5$.

Root loci and step responses for Problem 37

38. Consider the 270-ft U.S. Coast Guard cutter *Tampa* (902) shown in Fig. 5.67. Parameter identification based on sea-trials data (Trankle, 1987) was used to estimate the hydrodynamic coefficients in the equations of motion. The result is that the response of the heading angle of the ship ψ to rudder angle δ and wind changes w can be described by the second-order transfer functions

$$G_\delta(s) = \frac{\psi(s)}{\delta(s)} = \frac{-0.0184(s + 0.0068)}{s(s + 0.2647)(s + 0.0063)},$$

$$G_w(s) = \frac{\psi(s)}{w(s)} = \frac{0.0000064}{s(s + 0.2647)(s + 0.0063)},$$

where

$$\psi = \text{ heading angle, rad}$$
$$\psi_r = \text{reference heading angle, rad.}$$
$$r = \dot{\psi} \text{ yaw rate, rad/sec,}$$
$$\delta = \text{rudder angle, rad,}$$
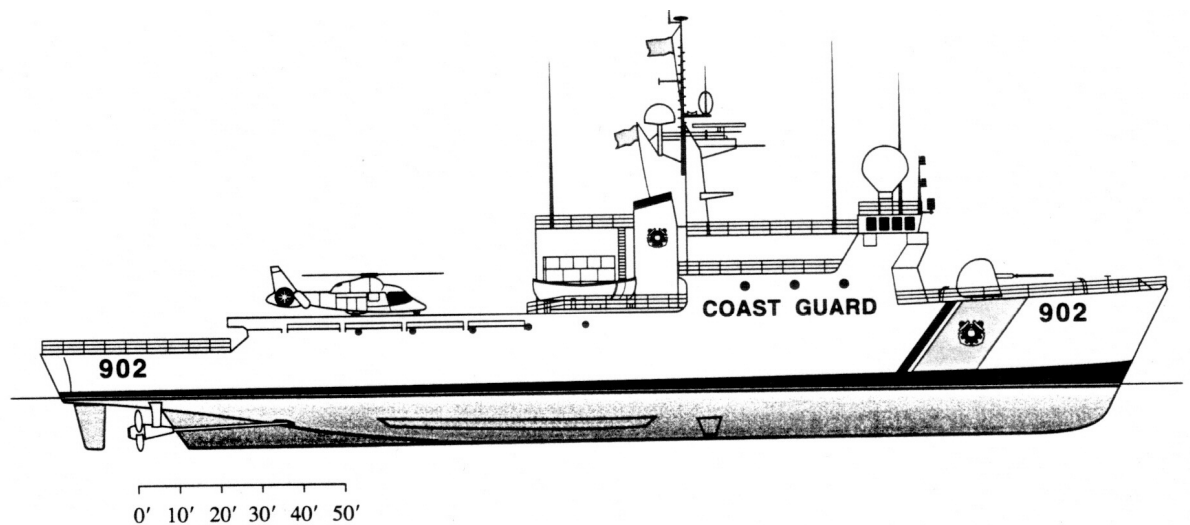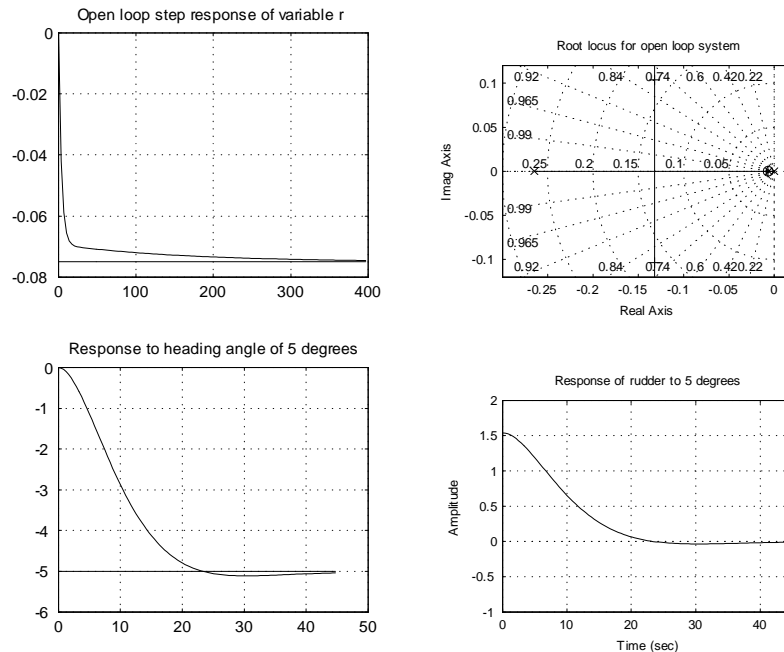$$w = \text{wind speed, m/sec.}$$

Figure 5.67: USCG cutter *Tampa* (902)

(a) Determine the open-loop settling time of r for a step change in δ.

(b) In order to regulate the heading angle ψ , design a compensator that uses ψ and the measurement provided by a yaw-rate gyroscope (that is, by $\dot{\psi} = r$). The settling time of ψ to a step change in ψ_r is specified to be less than 50 *sec*, and, for a 5° change in heading the maximum allowable rudder angle deflection is specified to be less than 10°.

(c) Check the response of the closed-loop system you designed in part (b) to a wind gust disturbance of 10 *m/sec* (model the disturbance as a step input). If the *steady-state* value of the heading due to this wind gust is more than 0.5°, modify your design so that it meets this specification as well.

Solution:

(a) From the transfer function final value theorem, the final value is
0.075. Using the step function in MATLAB, the settling time to 1%
of the final value is $t_s = 316.11$ sec.

(b) The maximum deflection of the rudder is almost surely at the initial
instant, when it is $\delta(0) = K\Psi_r(0)$. Thus to keep δ below $10°$ for a
step of $5°$, we need $K < 2.and$ for a settling time less than $50\,\sec$. we
need $\sigma > 4.6/50 = 0.092$. Drawing the root locus versus K and using
the function rlocfind we find that $K = 1.56$ gives roots with real
parts less than 0.13. The step response shows that this proportional
control is adequate for the problem.

(c) The steady-state error to a disturbance of $10m/\sec$ is less than 0.35.

Open loop step response of variable r

Root locus for open loop system

Response to heading angle of 5 degrees

Response of rudder to 5 degrees

39. Golden Nugget Airlines has opened a free bar in the tail of their airplanes
in an attempt to lure customers. In order to automatically adjust for the
sudden weight shift due to passengers rushing to the bar when it first
opens, the airline is mechanizing a pitch-attitude auto pilot. Figure 5.68
shows the block diagram of the proposed arrangement. We will model the
passenger moment as a step disturbance $M_p(s) = M_0/s$, with a maximum
expected value for M_0 of 0.6.

(a) What value of K is required to keep the steady-state error in θ to
less than 0.02 rad$(\cong 1°)$? (Assume the system is stable.)
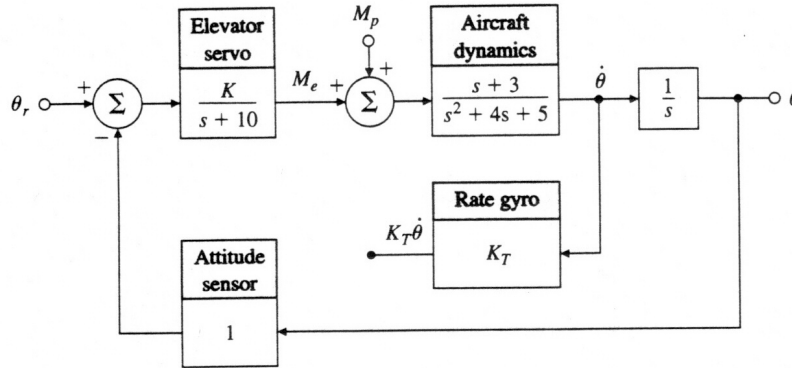
(b) Draw a root locus with respect to K.

Figure 5.68: Golden Nugget Airlines Autopilot

(c) Based on your root locus, what is the value of K when the system becomes unstable?

(d) Suppose the value of K required for acceptable steady-state behavior is 600. Show that this value yields an unstable system with roots at

$$s = -2.9, -13.5, +1.2 \pm 6.6j.$$

(e) You are given a black box with *rate gyro* written on the side and told that when installed, it provides a perfect measure of $\dot{\theta}$, with output $K_T\dot{\theta}$. Assume $K = 600$ as in part (d) and draw a block diagram indicating how you would incorporate the rate gyro into the auto pilot. (Include transfer functions in boxes.)

(f) For the rate gyro in part (e), sketch a root locus with respect to K_T.

(g) What is the maximum damping factor of the complex roots obtainable with the configuration in part (e)?

(h) What is the value of K_T for part (g)?

(i) Suppose you are not satisfied with the steady-state errors and damping ratio of the system with a rate gyro in parts (e) through (h). Discuss the advantages and disadvantages of adding an integral term and extra lead networks in the control law. Support your comments using MATLAB or with rough root-locus sketches.

Solution:

(a) $K = 300$.

(b) $K = 144$
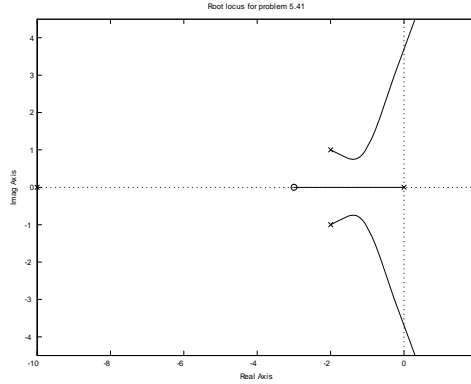
(c) The characteristic equation is $s^4 + 14s^3 + 45s^2 + 650s + 1800$. The exact roots are $-13.5, -2.94, -1.22 \pm 6.63$.

(d) The output of the rate gyro box would be added at the same spot as the attitude sensor output.

(e) $\zeta = 0.28$

(f) $K_T = 185/600 = 0.31$
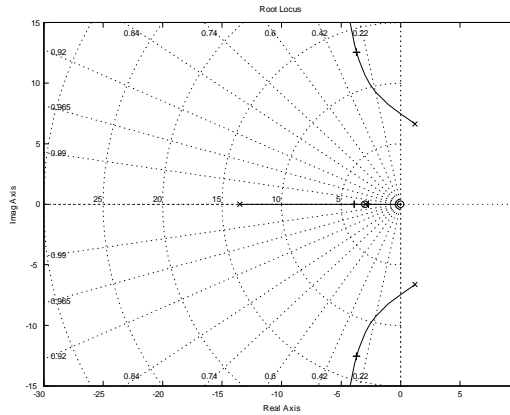
Root locus for problem 39f

(g) Integral (PI) control would reduce the steady-state error to the moment to zero but would make the damping less and the settling time longer. A lead network could improve the damping of the response.

40. Consider the instrument servomechanism with the parameters given in Fig. 5.69. For each of the following cases, draw a root locus with respect to the parameter K, and indicate the location of the roots corresponding to your final design.

   (a) *Lead network* : Let

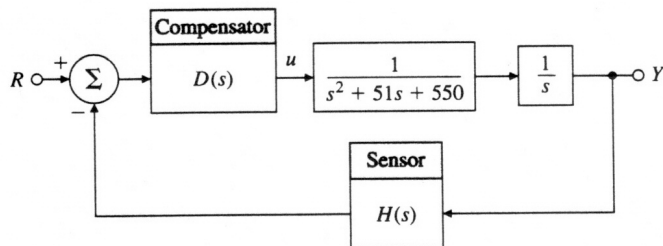$$H(s) = 1, \quad D(s) = K\frac{s+z}{s+p}, \quad \frac{p}{z} = 6.$$

Figure 5.69: Control system for problem 40

Select z and K so that the roots nearest the origin (the dominant roots) yield

$$\zeta \geq 0.4, \quad -\sigma \leq -7, \quad K_v \geq 16\frac{2}{3}\text{sec}^{-1}.$$

(b) *Output-velocity (tachometer) feedback*: Let

$$H(s) = 1 + K_T s \quad \text{and} \quad D(s) = K.$$

Select K_T and K so that the dominant roots are in the same location as those of part (a). Compute K_v. If you can, give a physical reason explaining the reduction in K_v when output derivative feedback is used.

(c) *Lag network* : Let

$$H(s) = 1 \quad \text{and} \quad D(s) = K\frac{s+1}{s+p}.$$

Using proportional control, it is possible to obtain a $K_v = 12$ at $\zeta = 0.4$. Select K and p so that the dominant roots correspond to the proportional-control case but with $K_v = 100$ rather than $K_v = 12$.

Solution:

(a) The K_v requirement leads to $K \geq 55000$. With this value, a root locus can be drawn with the parameter z by setting $p = 6z$.

$$1 + z\frac{\left[6s(s^2 + 51s + 550) + K\right]}{s^2(s^2 + 51s + 550) + Ks} = 0$$
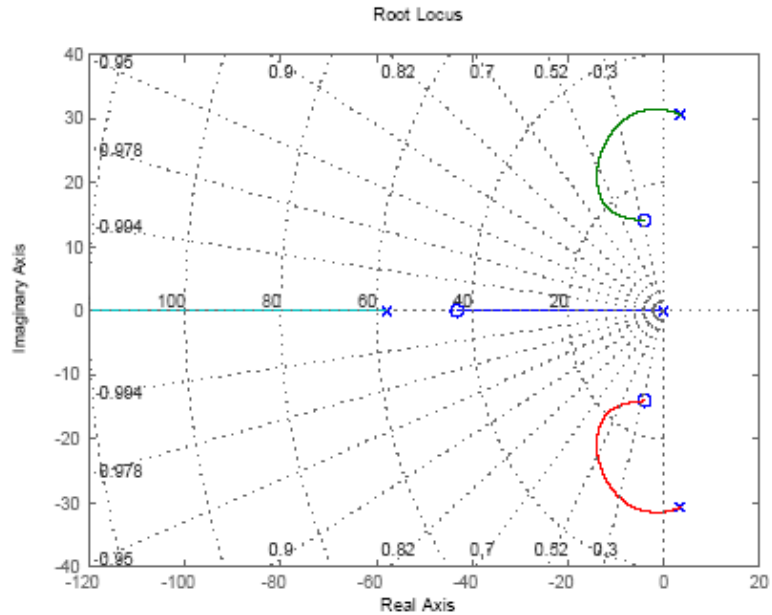
Root locus for Problem 5.40(a)

At the point of maximum damping, the values are $z = 17$ and the dominant roots are at about $-13 \pm j17$.

(b) To find the values of K and K_v, we compute a polynomial with roots at $-13 \pm j17$ and a third pole such that the coefficient of s^2 is 51,which is at $s = -25.15$ This calculation leads to $K = 11785$, $K_T = 0.0483$ and $K_v = 20.81$.

(c) The K_v needs to be increased by a factor of $100/12$. Thus, we have $p = 0.12$. The step responses of these designs are given in the plots below.
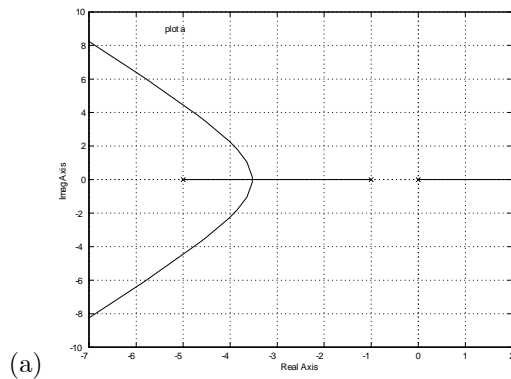
Step responses for problem 5.42
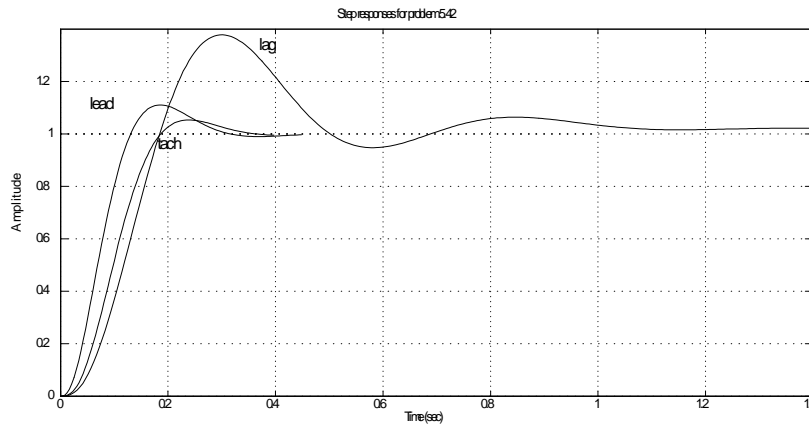
Problems and solutions for Section 5.6
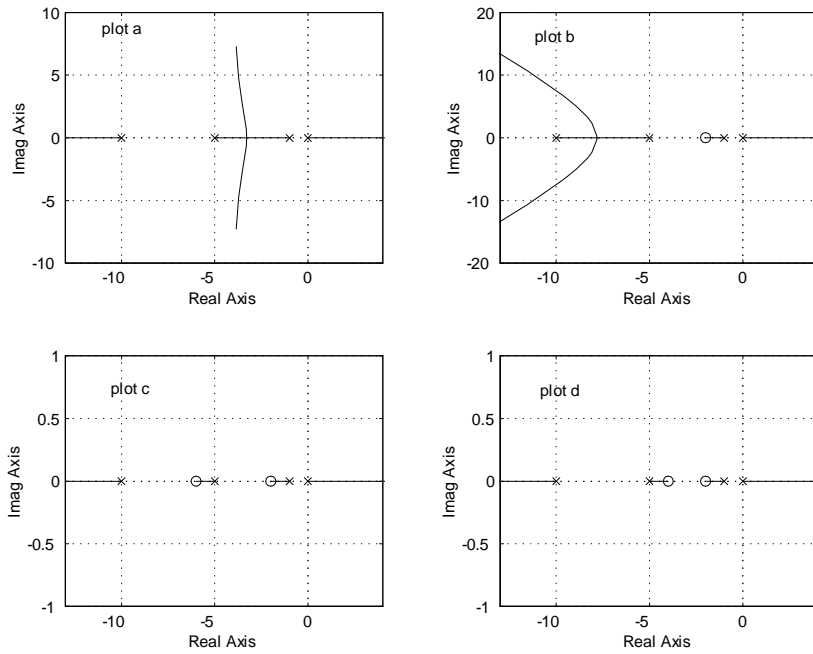
41. Plot the loci for the $0°$ locus or negative K for

   (a) The examples given in Problem 3
   (b) The examples given in Problem 4
   (c) The examples given in Problem 5
   (d) The examples given in Problem 6
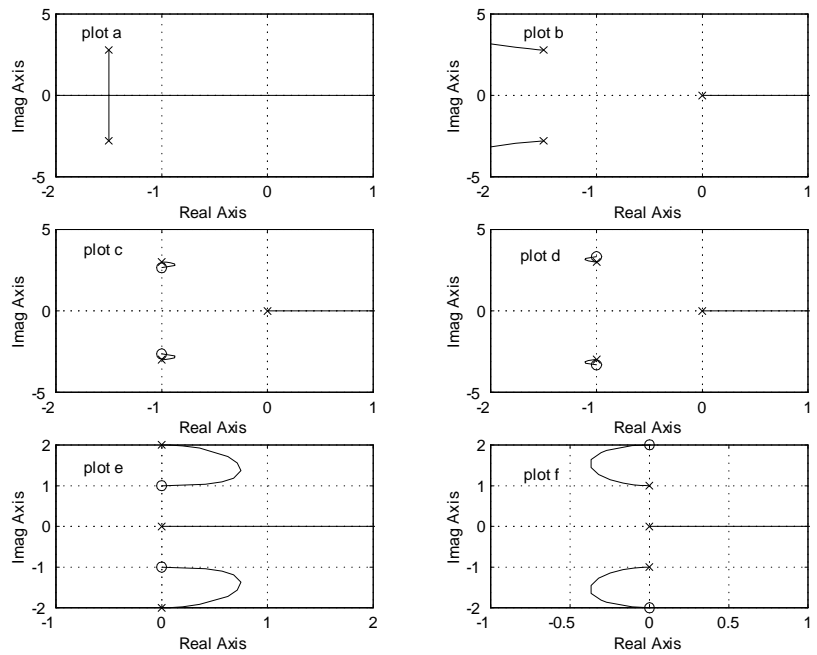   (e) The examples given in Problem 7
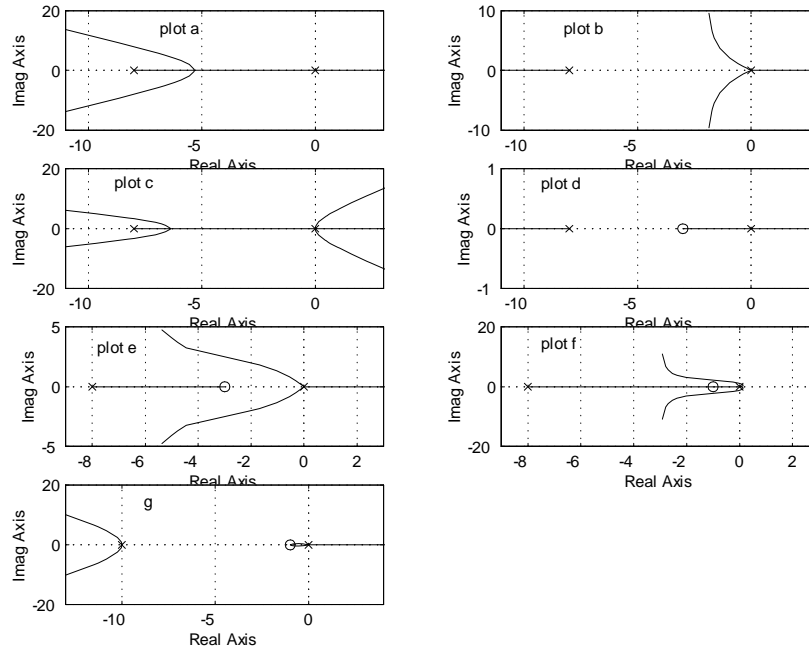   (f) The examples given in Problem 8

   Solution:

(a)

Problem 41(a)

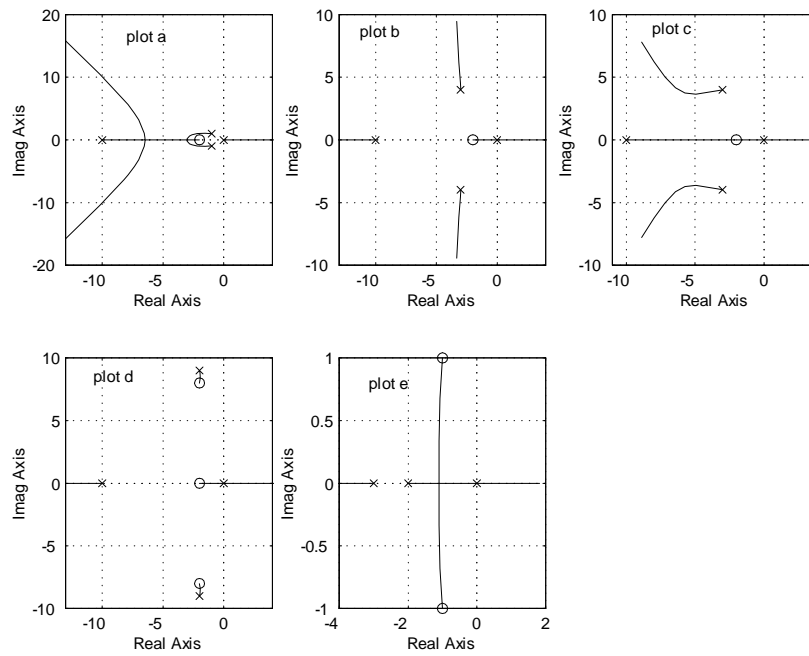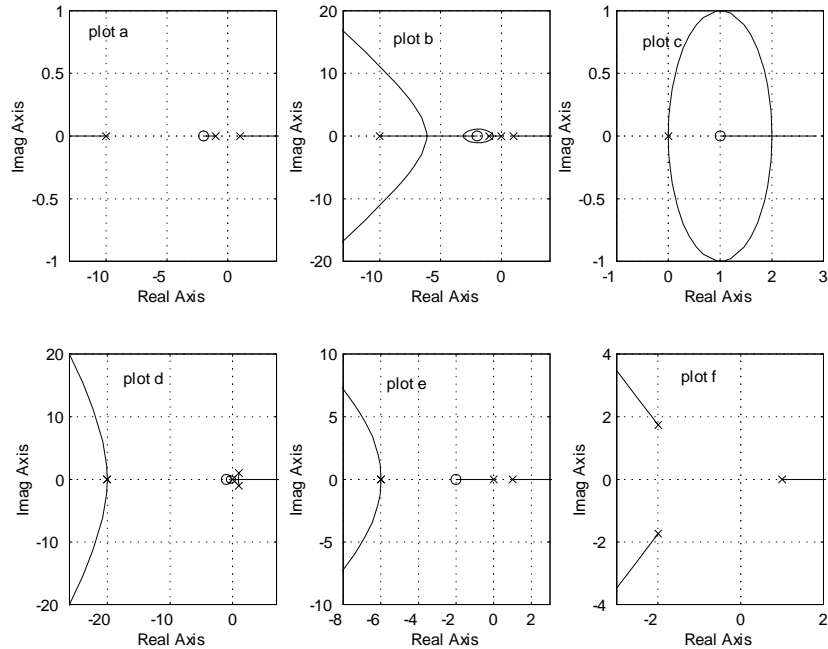Problem 41(b)

Problem 41(c)

Problem 41(d)

Problem 41(e)

Problem 41(f)

42. Suppose you are given the plant

$$L(s) = \frac{1}{s^2 + (1+\alpha)s + (1+\alpha)},$$

where α is a system parameter that is subject to variations. Use both positive and negative root-locus methods to determine what variations in α can be tolerated before instability occurs.

Solution:

$L(s) = \dfrac{s+1}{s^2 + s + 1}$.   the system is stable for all $\alpha > -1$. The complete locus is a circle of radius 1 centered on $s = -1$.

43. Consider the system in Fig. 5.70.

   (a) Use Routh's criterion to determine the regions in the (K_1, K_2) plane for which the system is stable.

   (b) Use rltool to verify your answer to part (a).
       Solution:

   (a) Define $k_p = K_1$ and $k_I = K_1 K_2$ and the characteristic equation is

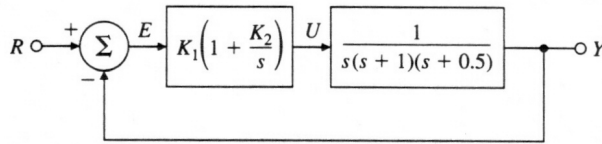$$s^4 + 1.5s^3 + 0.5s^2 + k_p s + k_I = 0$$

Figure 5.70: Feedback system for Problem 43

For this equation, Routh's criterion requires $k_I > 0$; $k_p < 0.75$; and $4k_p^2 - 3k_p + 9k_I < 0$. The third of these represents a parabola in the $[k_p, k_I]$ plane plotted below. The region of stability is the area under the parabola and above the k_p axis.

Stability region for problem 5.43

(b) When $k_I = 0$, there is obviously a pole at the origin. For points on the parabola, consider $k_p = 3/8$ and $k_I = 1/16$. The roots of the characteristic equation are -1.309, -0.191, and $\pm j0.5$.

44. The block diagram of a positioning servomechanism is shown in Fig. 5.71.

(a) Sketch the root locus with respect to K when no tachometer feedback is present $(K_T = 0)$.

(b) Indicate the root locations corresponding to $K = 16$ on the locus of part (a). For these locations, estimate the transient-response parameters t_r, M_p, and t_s. Compare your estimates to measurements obtained using the step command in MATLAB.

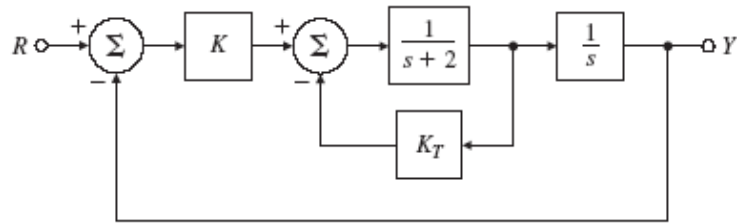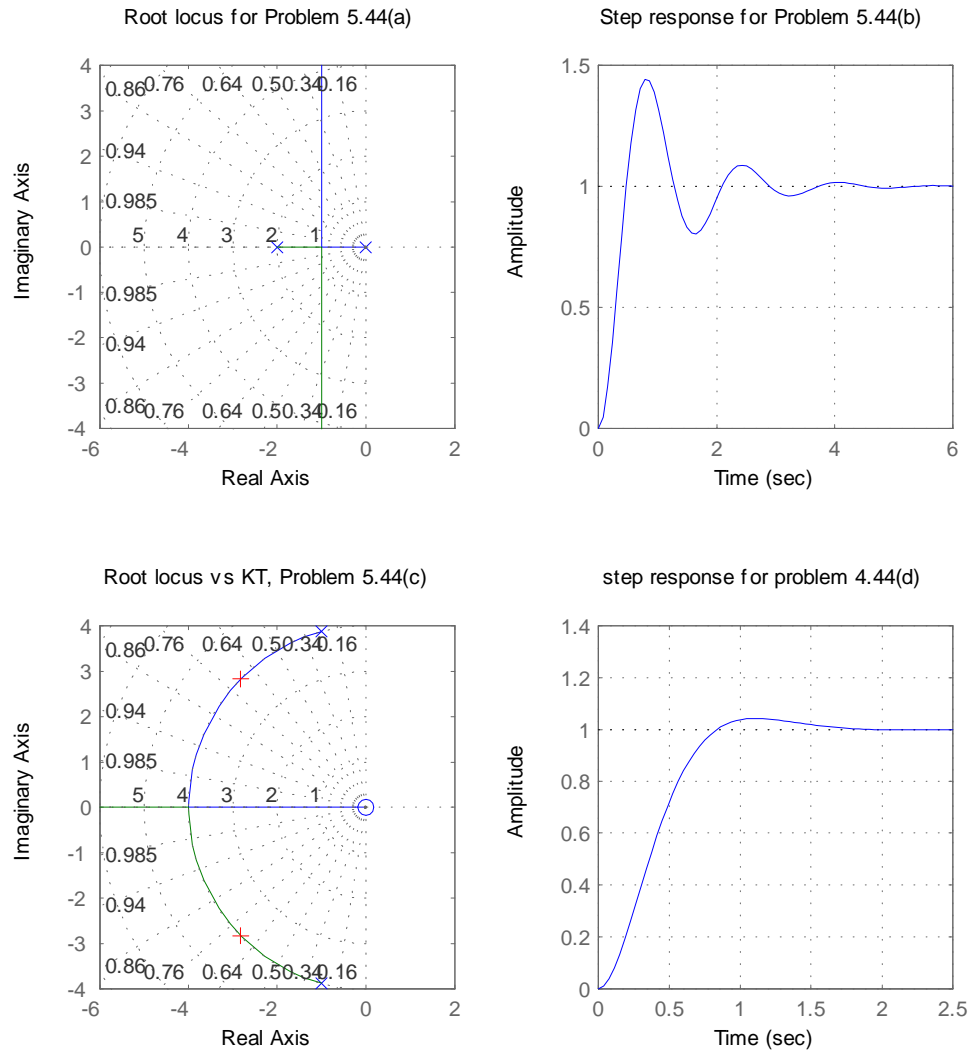(c) For $K = 16$, draw the root locus with respect to K_T.

Figure 5.71: Control system for problem 44

(d) For $K = 16$ and with K_T set so that $M_p = 0.05 (\zeta = 0.707)$, estimate t_r and t_s. Compare your estimates to the actual values of t_r and t_s obtained using MATLAB.

(e) For the values of K and K_T in part (d), what is the velocity constant K_v of this system?

Solution:

(a) The locus is the cross centered at $s = -0.5$

(b) The roots have a damping of 0.25 and natural frequency of 4. We'd estimate the overshoot to be $M_p = 45\%$ and a rise time of less than 0.45 sec. and settling time more than 4.6 sec. The values from the plot are approximately: $t_r = 0.4$, $M_p = 45\%$, and $t_s = 5\,\mathrm{sec}$. Not too bad.

(c) See below.

(d) Use rlocfind on the locus vs Kt to find the Kt value that yields 0.7 damping. This shows that $K_T = 3.66$. Using the formulas inside the back cover yields $M_p = 0.05$, $t_r = 0.45$, and $t_s = 1.6$.

Root locus for Problem 5.44(a)

Step response for Problem 5.44(b)

Root locus vs KT, Problem 5.44(c)

step response for problem 4.44(d)

Plots for problem 44

(e) Applying Eq. (4.33), we see that $K_v = K/(K_t + 2) = 2.83$.

45. Consider the mechanical system shown in Fig. 5.72, where g and a_0 are gains. The feedback path containing gs controls the amount of rate feedback. For a fixed value of a_0, adjusting g corresponds to varying the location of a zero in the s-plane.

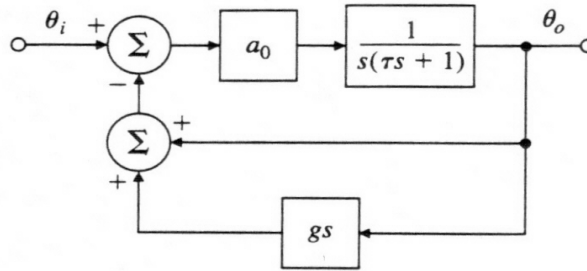(a) With $g = 0$ and $\tau = 1$, find a value for a_0 such that the poles are

Figure 5.72: Control system for problem 5.46

complex.

(b) Fix a_0 at this value, and construct a root locus that demonstrates the effect of varying g.

Solution:

(a) The roots are complex for $a_0 > 0.25$. We select $a_0 = 1$ and the roots are at $s = -0.5 \pm 0.866$

(b) With respect to g, the root locus equation is $s^2 + s + 1 + gs = 0$. The locus is a circle, plotted below.
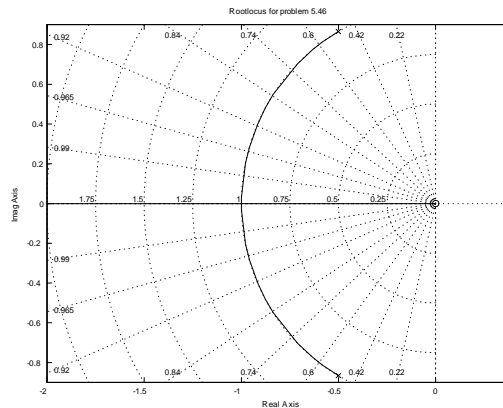
46. Sketch the root locus with respect to K for the system in Fig. 5.73. What is the range of values of K for which the system is unstable?

Solution:

MATLAB cannot directly plot a root locus for a transcendental function. However, with the Pade' approximation, a locus valid for small values of s can be plotted, as shown below.
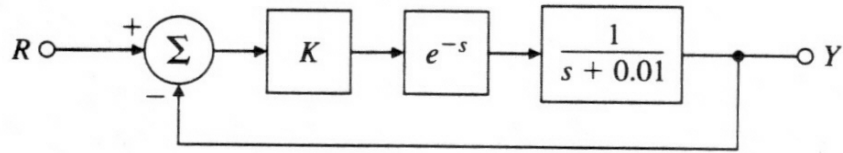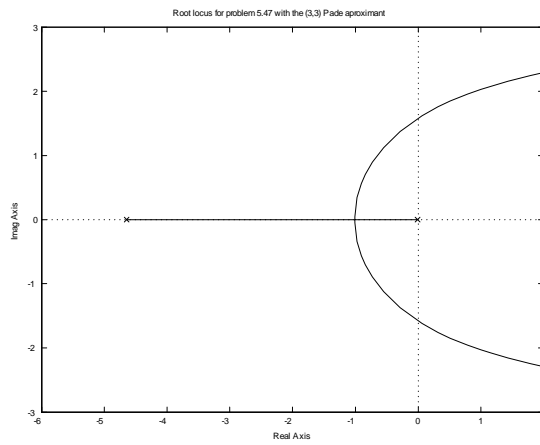
Figure 5.73: Control system for problem 5.46

Root locus for problem 5.47 with the (3,3) Pade aproximant

Solutions for problem 5.46

47. Prove that the plant $G(s) = 1/s^3$ *cannot* be made unconditionally stable if pole cancellation is forbidden.

Solution:

The angles of departure from a triple pole are 180 and ±60 for the negative locus and 0 and ±120 for the positive locus. In either case, at least one pole starts out into the right-half plane. Such a system must be conditionally stable for it will be unstable if the gain is small enough.
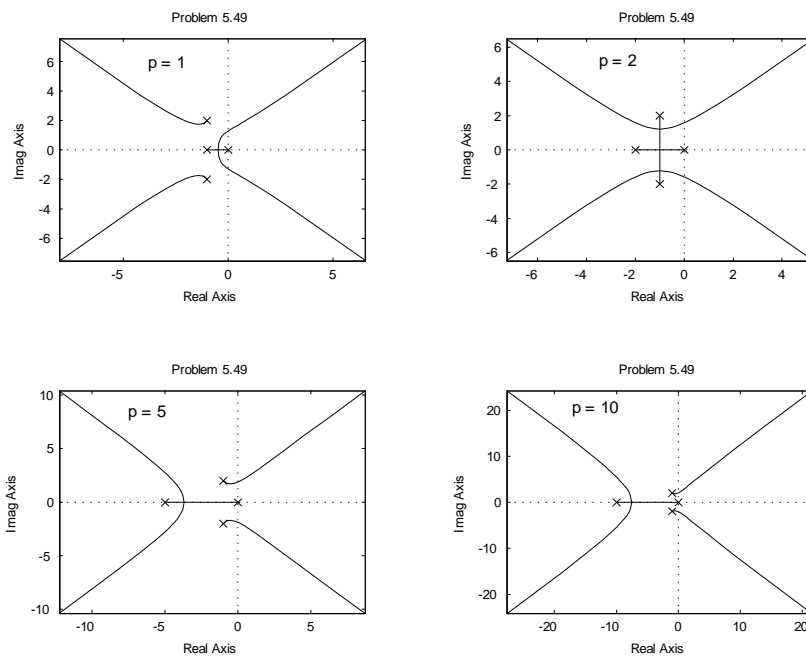
48. For the equation $1 + KG(s)$ where

$$G(s) = \frac{1}{s(s+p)[(s+1)^2 + 4]},$$

use MATLAB to examine the root locus as a function of K for p in the range from $p = 1$ to $p = 10$, making sure to include the point $p = 2$.

Solution:

The root loci for four values are given in the figure. The point is that the locus for $p = 2$ has multiple roots at a complex value of s.

Solutions for problem 48