TE055

Lugar das Raízes: exercícios

Profª Juliana L. M. Iamamura

Exercício 1

Esboce o lugar das raízes do sistema abaixo para $0 \leq K < \infty$.

$$KG(s) = \frac{K}{s(s+4)(s^2+8s+32)}$$

Exercício 2

Dado o sistema representado pela FTMA abaixo:

(a) Determine a faixa de valores de K para a qual o sistema em malha fechada seja estável.

(b) Esboce o lugar das raízes para $0 \leq K < \infty$.

(c) Determine o valor de K para o qual a função de transferência de malha fechada tenha um polo em s = -2.

$$G(s) = \frac{1}{s\left(s^2 + 4\text{s} + 5\right)}$$

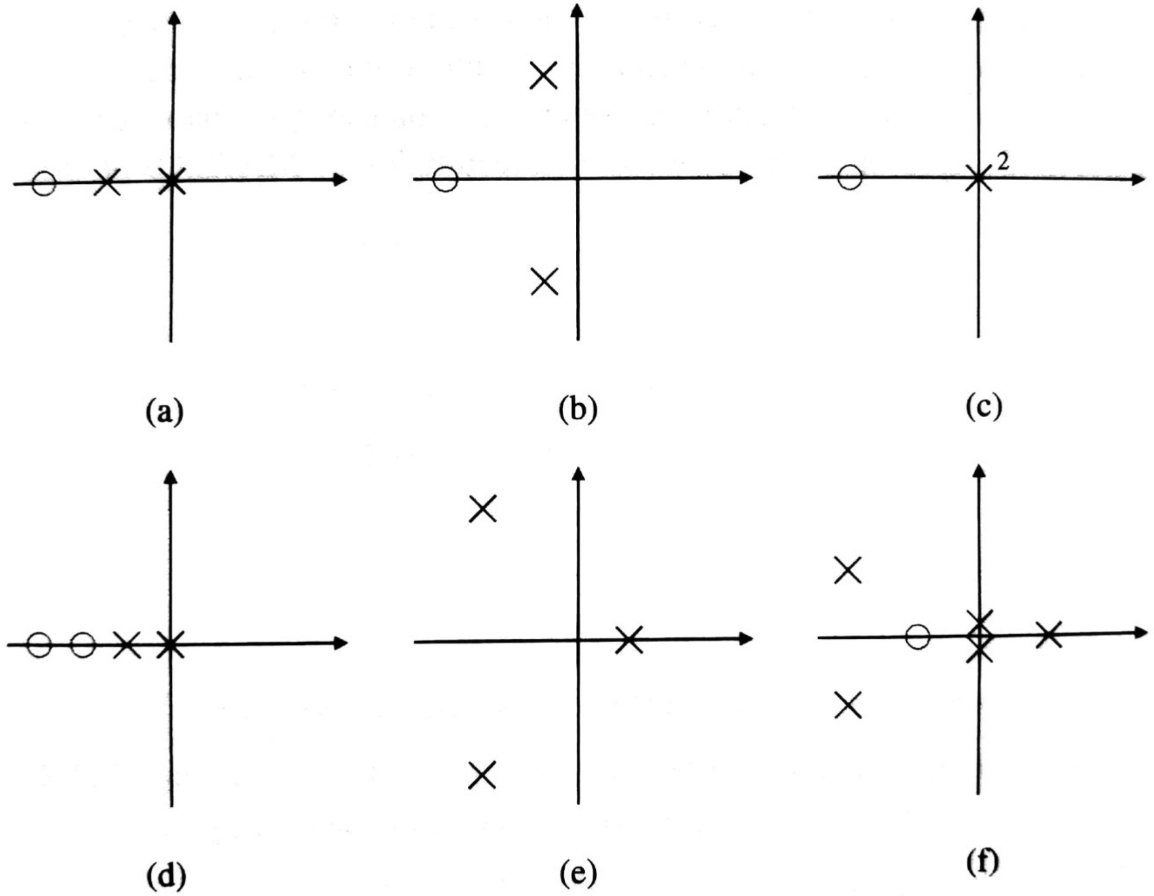
Exercício 3

2. Roughly sketch the root loci for the pole-zero maps as shown in Fig. 5.51. Show your estimates of the center and angles of the asymptotes, a rough evaluation of arrival and departure angles for complex poles and zeros, and the loci for positive values of the parameter K. Each pole-zero map is from a characteristic equation of the form

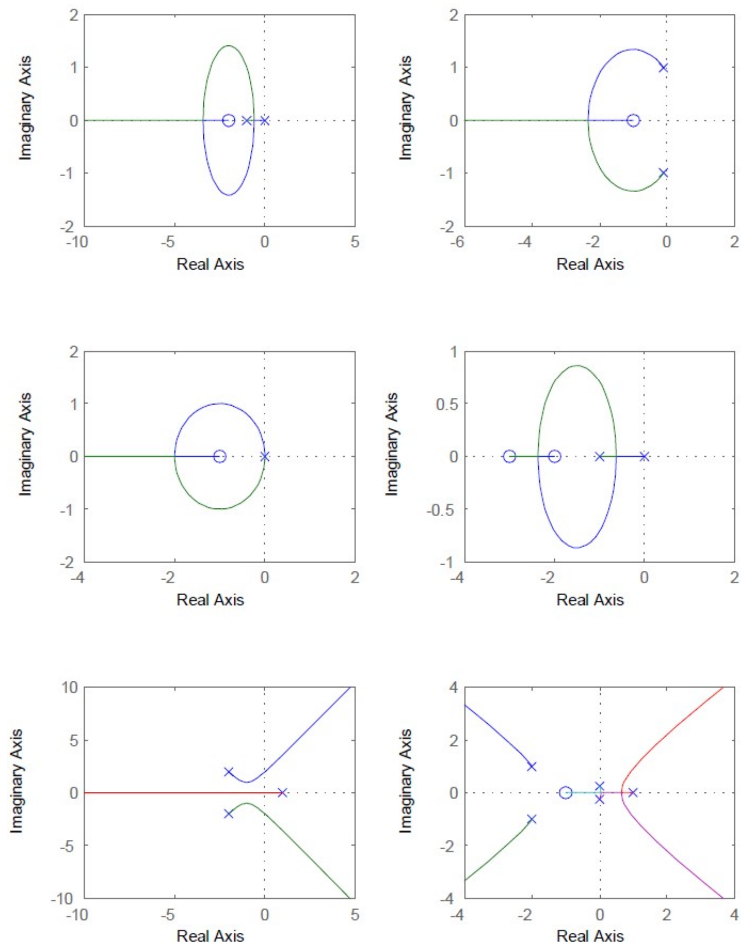
$$1 + K\frac{b(s)}{a(s)} = 0,$$

where the roots of the numerator $b(s)$ are shown as small circles o and the roots of the denominator $a(s)$ are shown as $\times's$ on the s-plane. Note that in Fig. 5.51(c), there are two poles at the origin and there are two poles on the imaginary axis in (f), slightly off the real axis.

Exercício 3



Exercício 3 - solução



Exercício 4

3. For the characteristic equation

$$1 + \frac{K}{s(s+1)(s+5)} = 0:$$

(a) Draw the real-axis segments of the corresponding root locus.

(b) Sketch the asymptotes of the locus for $K \to \infty$.

(c) For what value of K are the roots on the imaginary axis?

(d) Verify your sketch with a MATLAB plot.

Exercício 5

5.4 *Polos e zeros reais*. Esboce o lugar das raízes com respeito a K para a equação $1 + KL(s) = 0$ com as escolhas de $L(s)$ listadas. Certifique-se de apresentar as assíntotas e os ângulos de chegada e partida em qualquer zero ou polo complexo. Depois de completar cada esboço à mão, verifique os resultados usando o MATLAB. Apresente seus esboços e os resultados do MATLAB na mesma escala.

(a) $L(s) = \dfrac{(s+2)}{s(s+1)(s+5)(s+10)}$

(b) $L(s) = \dfrac{1}{s(s+1)(s+5)(s+10)}$

(c) $L(s) = \dfrac{(s+2)(s+6)}{s(s+1)(s+5)(s+10)}$

(d) $L(s) = \dfrac{(s+2)(s+4)}{s(s+1)(s+5)(s+10)}$

Exercício 6

5.5 *Polos e zeros complexos*. Esboce o lugar das raízes com respeito a K para a equação $1 + KL(s) = 0$ com as escolhas de $L(s)$ listadas. Certifique-se de apresentar as assíntotas e os ângulos de chegada e partida em qualquer zero ou polo complexo. Depois de completar cada esboço à mão, verifique os resultados usando o MATLAB. Apresente seus esboços e os resultados do MATLAB na mesma escala.

(a) $L(s) = \dfrac{1}{s^2 + 3s + 10}$

(b) $L(s) = \dfrac{1}{s(s^2 + 3s + 10)}$

(c) $L(s) = \dfrac{(s^2 + 2s + 8)}{s(s^2 + 2s + 10)}$

(d) $L(s) = \dfrac{(s^2 + 2s + 12)}{s(s^2 + 2s + 10)}$

(e) $L(s) = \dfrac{(s^2 + 1)}{s(s^2 + 4)}$

(f) $L(s) = \dfrac{(s^2 + 4)}{s(s^2 + 1)}$

Exercício 7

6. *Multiple poles at the origin* Sketch the root locus with respect to K for the equation $1 + KL(s) = 0$ and the following choices for $L(s)$. Be sure to give the asymptotes, arrival and departure angles at any complex zero or pole, and the frequency of any imaginary-axis crossing. After completing each hand sketch verify your results using MATLAB. Turn in your hand sketches and the MATLAB results on the same scales.

(a) $L(s) = \dfrac{1}{s^2(s+8)}$

(b) $L(s) = \dfrac{1}{s^3(s+8)}$

(c) $L(s) = \dfrac{1}{s^4(s+8)}$

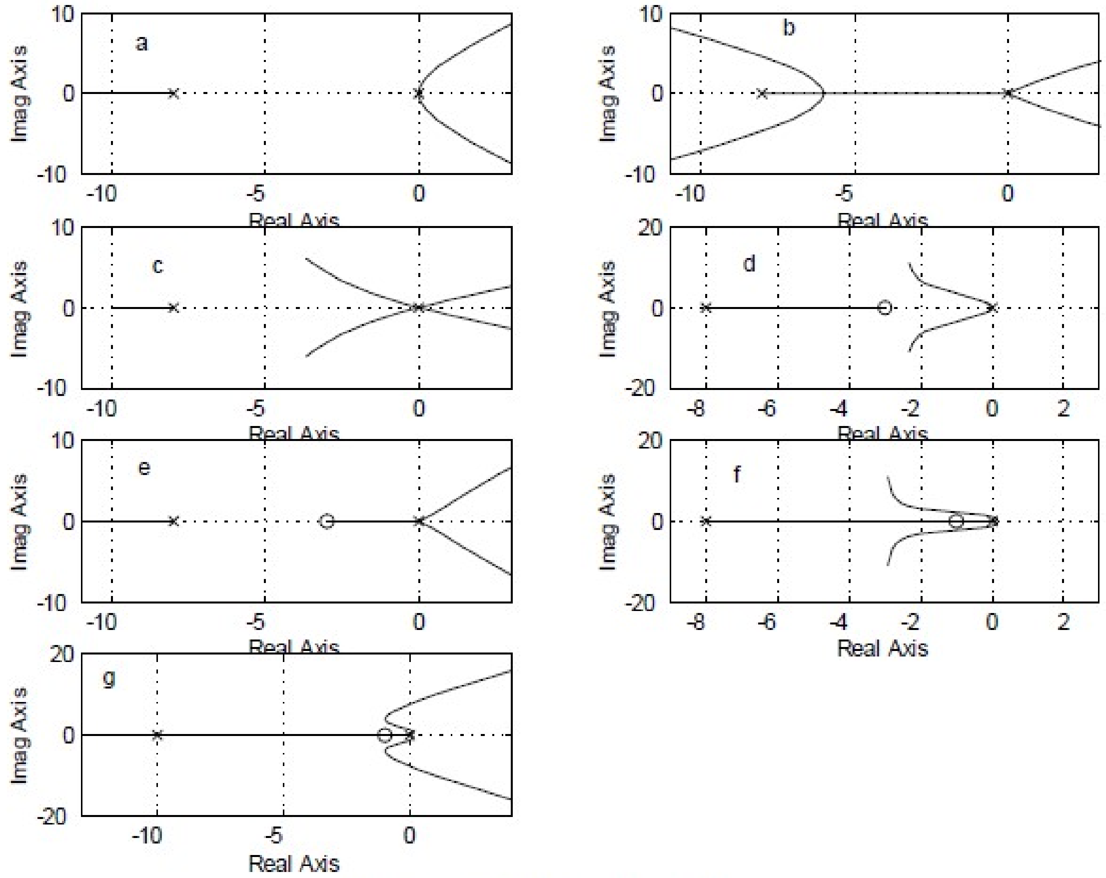
(d) $L(s) = \dfrac{(s+3)}{s^2(s+8)}$

(e) $L(s) = \dfrac{(s+3)}{s^3(s+4)}$

(f) $L(s) = \dfrac{(s+1)^2}{s^3(s+4)}$

(g) $L(s) = \dfrac{(s+1)^2}{s^3(s+10)^2}$

Exercício 7 - Solução



Solution for Problem 5.6

Exercício 8

7. *Mixed real and complex poles* Sketch the root locus with respect to K for the equation $1 + KL(s) = 0$ and the following choices for $L(s)$. Be sure to give the asymptotes, arrival and departure angles at any complex zero or pole, and the frequency of any imaginary-axis crossing. After completing each hand sketch verify your results using MATLAB. Turn in your hand sketches and the MATLAB results on the same scales.

(a) $L(s) = \dfrac{(s+2)}{s(s+10)(s^2+2s+2)}$

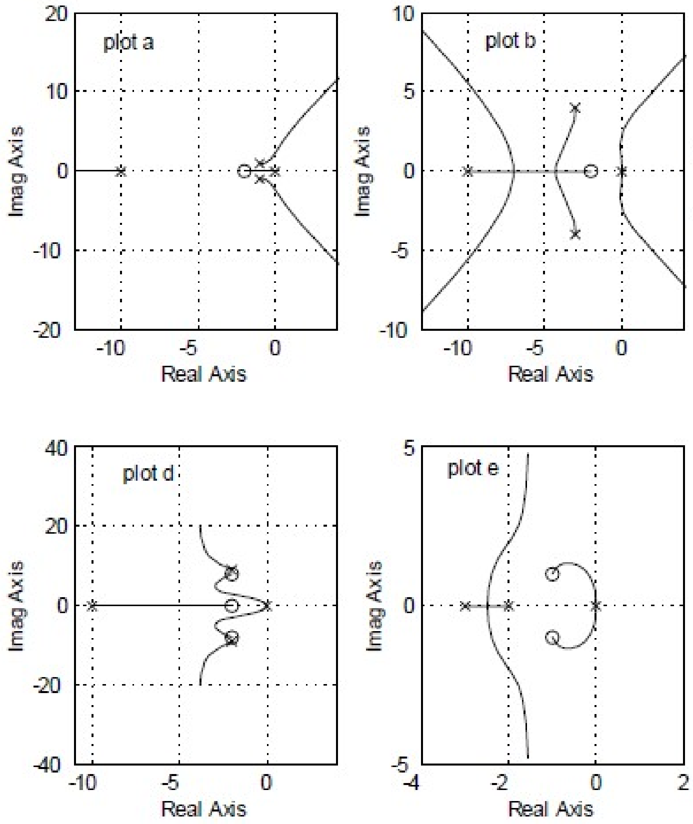
(b) $L(s) = \dfrac{(s+2)}{s^2(s+10)(s^2+6s+25)}$

(c) $L(s) = \dfrac{(s+2)^2}{s^2(s+10)(s^2+6s+25)}$

(d) $L(s) = \dfrac{(s+2)(s^2+4s+68)}{s^2(s+10)(s^2+4s+85)}$

(e) $L(s) = \dfrac{[(s+1)^2+1]}{s^2(s+2)(s+3)}$

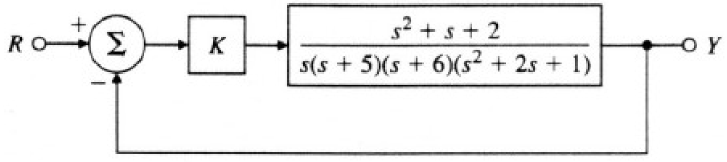
Exercício 8 - Solução



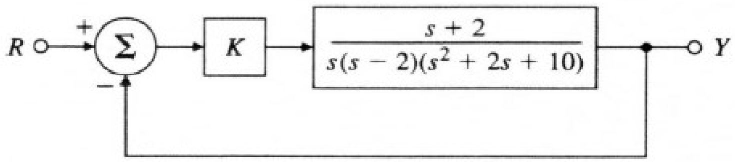
Solution for Problem 5.7

Exercício 9

11. Use the Routh criterion to find the range of the gain K for which the systems in Fig. 5.53 are stable and use the root locus to confirm your calculations.



(a)



(b)

Figure 5.53: Feedback systems for problem 11

Exercício 10

15. A simplified model of the longitudinal motion of a certain helicopter near hover has the transfer function

$$G(s) = \frac{9.8(s^2 - 0.5s + 6.3)}{(s + 0.66)(s^2 - 0.24s + 0.15)}.$$

and the characteristic equation $1 + D(s)G(s) = 0$. Let $D(s) = k_p$ at first.

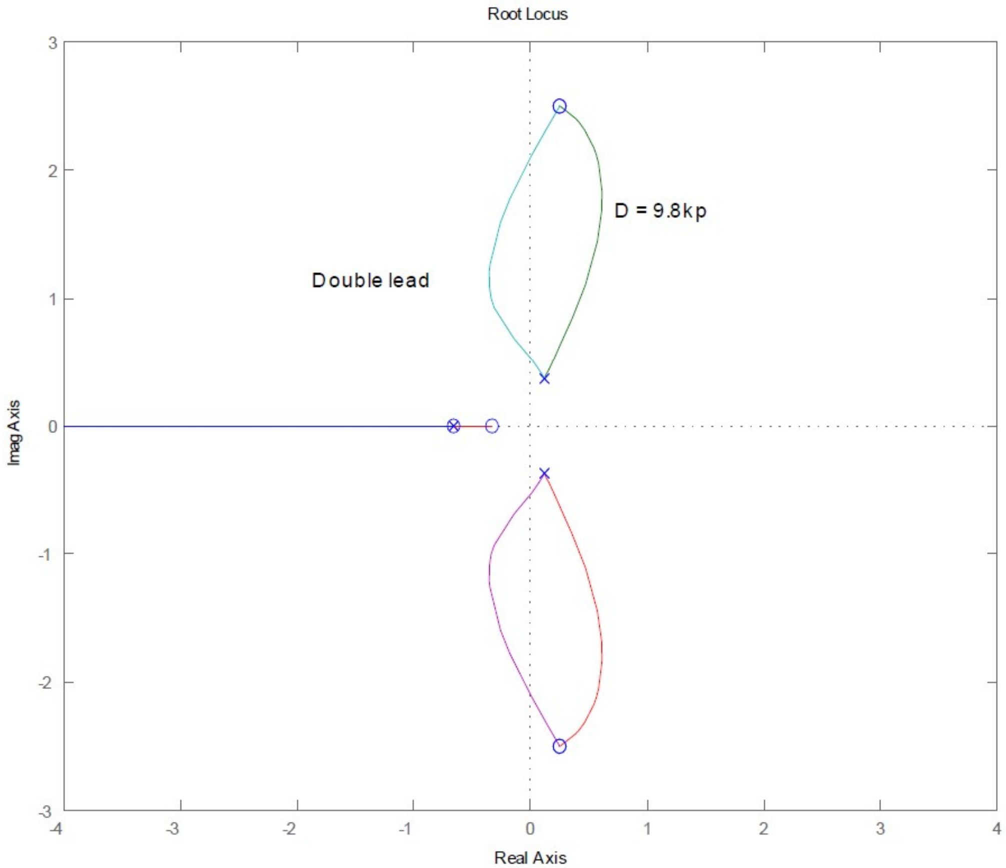
(a) Compute the departure and arrival angles at the complex poles and zeros.

(b) Sketch the root locus for this system for parameter $K = 9.8k_p$.Use axes -4 $\leq x \leq 4$. $-3 \leq y \leq 3$;

(c) Verify your answer using MATLAB. Use the command axis([-4 4 -3 3]) to get the right scales.

(d) Suggest a practical (at least as many poles as zeros) alternative compensation $D(s)$ which will at least result in a stable system.

Exercício 10 - LR



Exercício 11

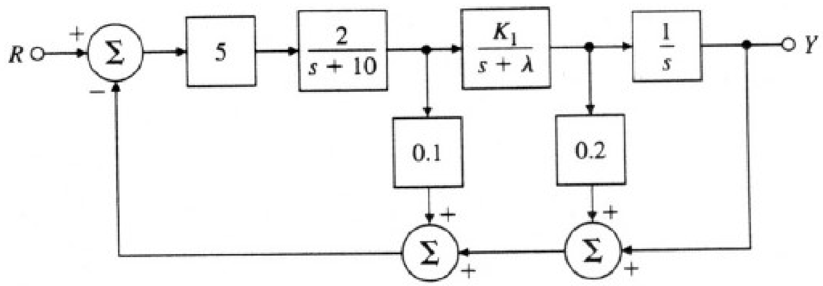


Figure 5.56: Control system for problem 5.16

16. For the system given in Fig. 5.56,

(a) plot the root locus of the characteristic equation as the parameter K_1 is varied from 0 to ∞ with $\lambda = 2$. Give the corresponding $L(s)$, $a(s)$, and $b(s)$.

(b) Repeat part (a) with $\lambda = 5$. Is there anything special about this value?

(c) Repeat part (a) for fixed $K_1 = 2$ with the parameter $K = \lambda$ varying from 0 to ∞.

Exercício 12

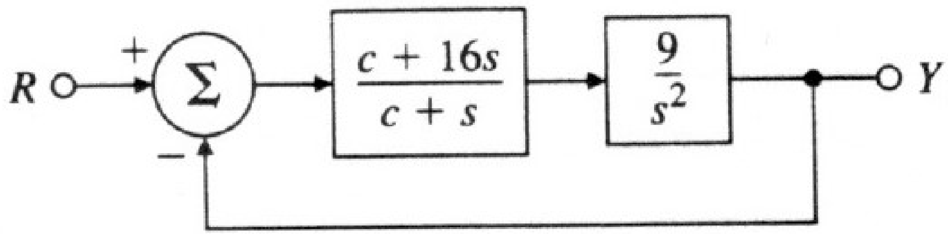


Figure 5.57: Control system for problem 17

17. For the system shown in Fig. 5.57, determine the characteristic equation and sketch the root locus of it with respect to positive values of the parameter c. Give $L(s)$, $a(s)$, and $b(s)$ and be sure to show with arrows the direction in which c increases on the locus.

13.43. Determine the angle and magnitude of

$$GH = \frac{20(s+10+j10)(s+10-j10)}{(s+10)(s+15)(s+25)}$$

at the following points in the s-plane: (*a*) $s = j10$, (*b*) $s = j20$, (*c*) $s = -10 + j20$, (*d*) $s = -20 + j20$, (*e*) $s = -15 + j5$.

13.44. For each transfer function, find the breakaway points on the root-locus:

(*a*) $GH = \dfrac{K}{s(s+6)(s+8)}$, (*b*) $GH = \dfrac{K(s+5)}{(s+2)(s+4)}$, (*c*) $GH = \dfrac{K(s+1)}{s^2(s+9)}$.

13.45. Find the departure angle of the root-locus from the pole at $s = -10 + j10$ for

$$GH = \frac{K(s+8)}{(s+14)(s+10+j10)(s+10-j10)} \qquad K > 0$$

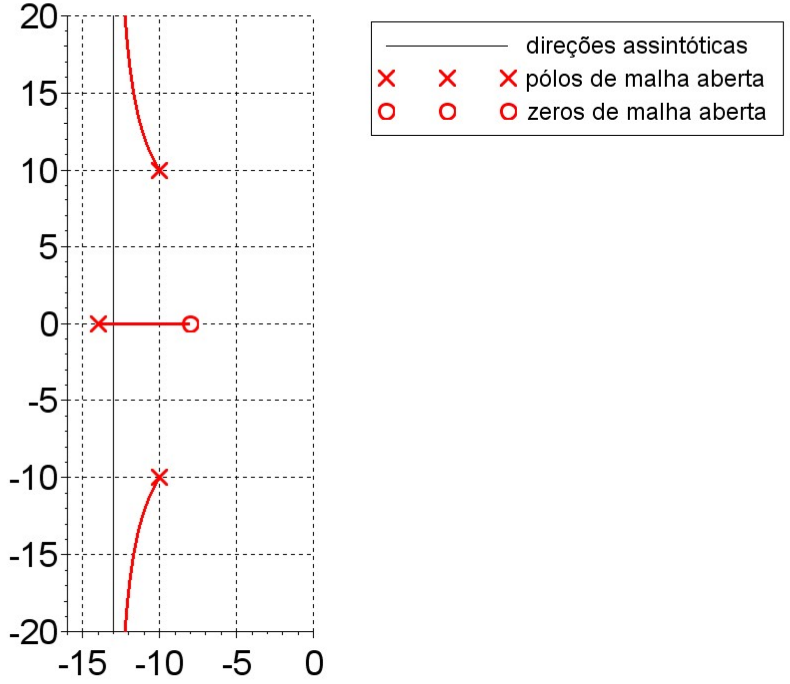
13.46. Find the departure angle of the root-locus from the pole at $s = -15 + j9$ for

$$GH = \frac{K}{(s+5)(s+10)(s+15+j9)(s+15-j9)} \qquad K > 0$$

13.47. Find the arrival angle of the root-locus to the zero at $s = -7 + j5$ for

$$GH = \frac{K(s+7+j5)(s+7-j5)}{(s+3)(s+5)(s+10)} \qquad K > 0$$

13.45 – Lugar das raízes



13.45 – Lugar das raízes