TE055

Lugar das Raízes: exercícios

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Exercício 1

Esboce o lugar das raízes do sistema abaixo para

$0 \leq K < \infty$.

$$KG(s) = \frac{K}{s(s+4)(s^2+8s+32)}$$

Exercício 2

Dado o sistema representado pela FTMA abaixo:

(a) Determine a faixa de valores de K para a qual o sistema em malha fechada seja estável.

(b) Esboce o lugar das raízes para $0 \leq K < \infty$.

(c) Determine o valor de K para o qual a função de transferência de malha fechada tenha um polo em $s = -2$.

$$G(s) = \frac{1}{s(s^2 + 4s + 5)}$$

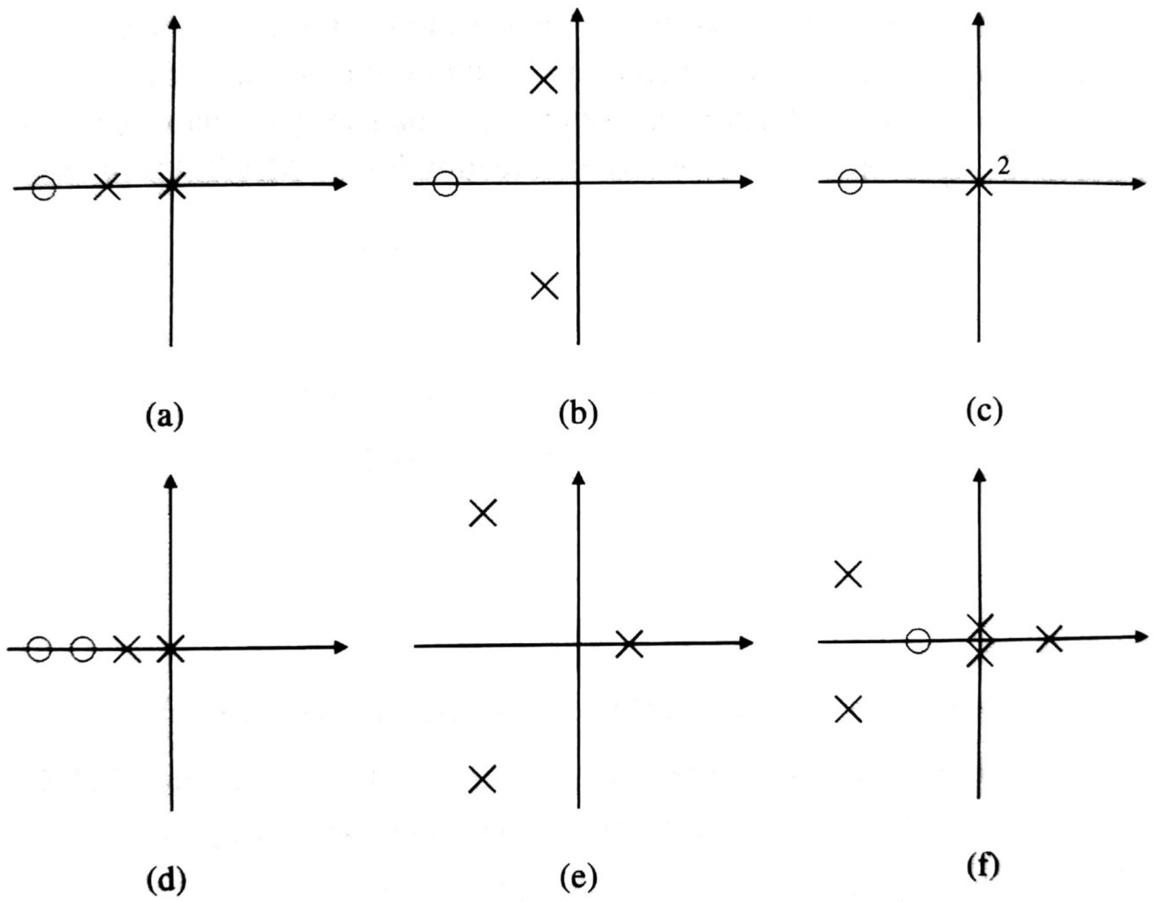
Exercício 3

2. Roughly sketch the root loci for the pole-zero maps as shown in Fig. 5.51. Show your estimates of the center and angles of the asymptotes, a rough evaluation of arrival and departure angles for complex poles and zeros, and the loci for positive values of the parameter K. Each pole-zero map is from a characteristic equation of the form

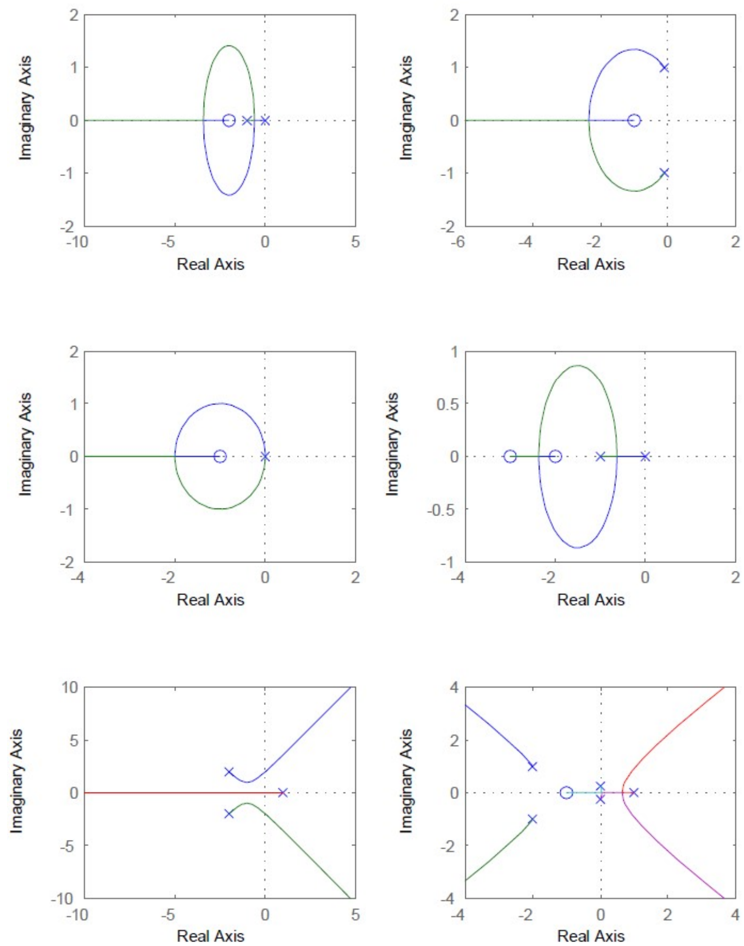
$$1 + K\frac{b(s)}{a(s)} = 0,$$

where the roots of the numerator $b(s)$ are shown as small circles o and the roots of the denominator $a(s)$ are shown as $\times' s$ on the s-plane. Note that in Fig. 5.51(c), there are two poles at the origin and there are two poles on the imaginary axis in (f), slightly off the real axis.

Exercício 3



Exercício 3 - solução



Exercício 4

3. For the characteristic equation

$$1 + \frac{K}{s(s+1)(s+5)} = 0:$$

(a) Draw the real-axis segments of the corresponding root locus.

(b) Sketch the asymptotes of the locus for $K \to \infty$.

(c) For what value of K are the roots on the imaginary axis?

(d) Verify your sketch with a MATLAB plot.

Exercício 5

5.4 *Polos e zeros reais*. Esboce o lugar das raízes com respeito a K para a equação $1 + KL(s) = 0$ com as escolhas de $L(s)$ listadas. Certifique-se de apresentar as assíntotas e os ângulos de chegada e partida em qualquer zero ou polo complexo. Depois de completar cada esboço à mão, verifique os resultados usando o MATLAB. Apresente seus esboços e os resultados do MATLAB na mesma escala.

(a) $L(s) = \dfrac{(s+2)}{s(s+1)(s+5)(s+10)}$

(b) $L(s) = \dfrac{1}{s(s+1)(s+5)(s+10)}$

(c) $L(s) = \dfrac{(s+2)(s+6)}{s(s+1)(s+5)(s+10)}$

(d) $L(s) = \dfrac{(s+2)(s+4)}{s(s+1)(s+5)(s+10)}$

Exercício 6

5.5 *Polos e zeros complexos*. Esboce o lugar das raízes com respeito a K para a equação $1 + KL(s) = 0$ com as escolhas de $L(s)$ listadas. Certifique-se de apresentar as assíntotas e os ângulos de chegada e partida em qualquer zero ou polo complexo. Depois de completar cada esboço à mão, verifique os resultados usando o MATLAB. Apresente seus esboços e os resultados do MATLAB na mesma escala.

(a) $L(s) = \dfrac{1}{s^2 + 3s + 10}$

(b) $L(s) = \dfrac{1}{s(s^2 + 3s + 10)}$

(c) $L(s) = \dfrac{(s^2 + 2s + 8)}{s(s^2 + 2s + 10)}$

(d) $L(s) = \dfrac{(s^2 + 2s + 12)}{s(s^2 + 2s + 10)}$

(e) $L(s) = \dfrac{(s^2 + 1)}{s(s^2 + 4)}$

(f) $L(s) = \dfrac{(s^2 + 4)}{s(s^2 + 1)}$

Exercício 7

6. *Multiple poles at the origin* Sketch the root locus with respect to K for the equation $1 + KL(s) = 0$ and the following choices for $L(s)$. Be sure to give the asymptotes, arrival and departure angles at any complex zero or pole, and the frequency of any imaginary-axis crossing. After completing each hand sketch verify your results using MATLAB. Turn in your hand sketches and the MATLAB results on the same scales.

(a) $L(s) = \dfrac{1}{s^2(s+8)}$

(b) $L(s) = \dfrac{1}{s^3(s+8)}$

(c) $L(s) = \dfrac{1}{s^4(s+8)}$

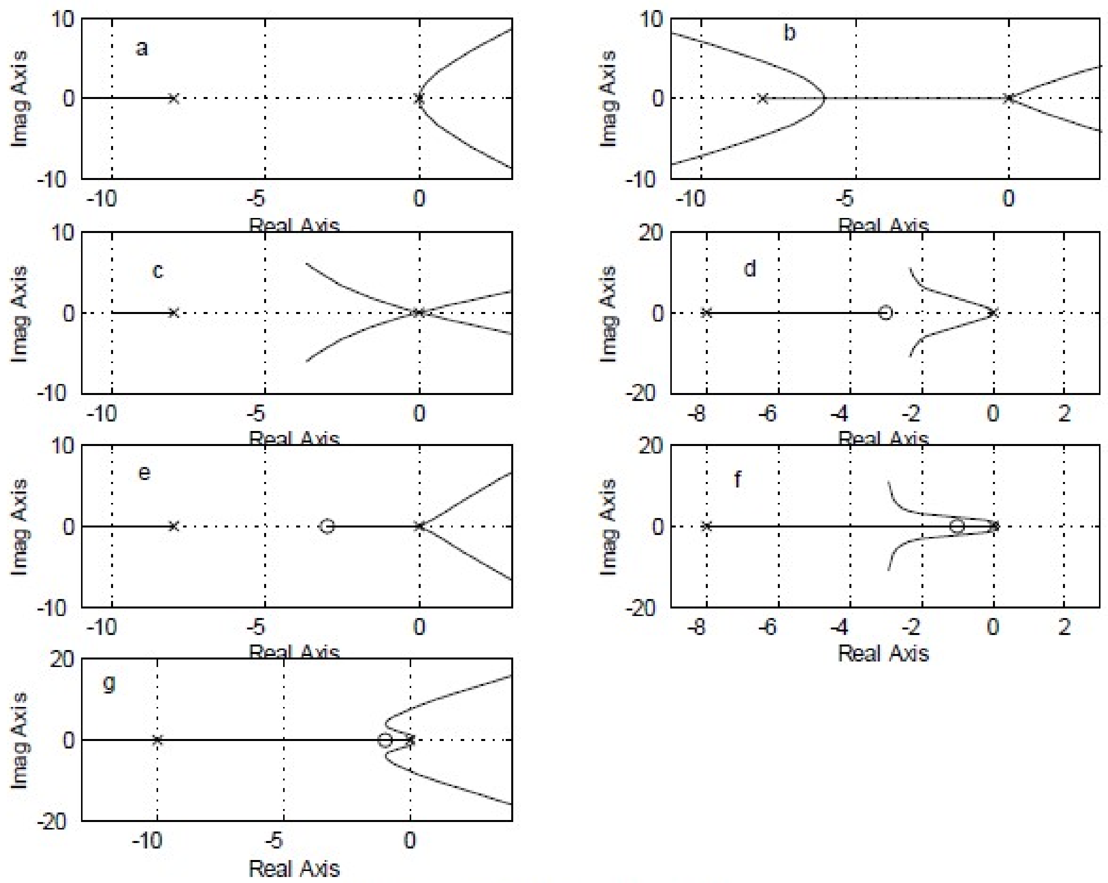
(d) $L(s) = \dfrac{(s+3)}{s^2(s+8)}$

(e) $L(s) = \dfrac{(s+3)}{s^3(s+4)}$

(f) $L(s) = \dfrac{(s+1)^2}{s^3(s+4)}$

(g) $L(s) = \dfrac{(s+1)^2}{s^3(s+10)^2}$

Exercício 7 - Solução



Solution for Problem 5.6

Exercício 8

7. *Mixed real and complex poles* Sketch the root locus with respect to K for the equation $1 + KL(s) = 0$ and the following choices for $L(s)$. Be sure to give the asymptotes, arrival and departure angles at any complex zero or pole, and the frequency of any imaginary-axis crossing. After completing each hand sketch verify your results using MATLAB. Turn in your hand sketches and the MATLAB results on the same scales.

(a) $L(s) = \dfrac{(s+2)}{s(s+10)(s^2+2s+2)}$

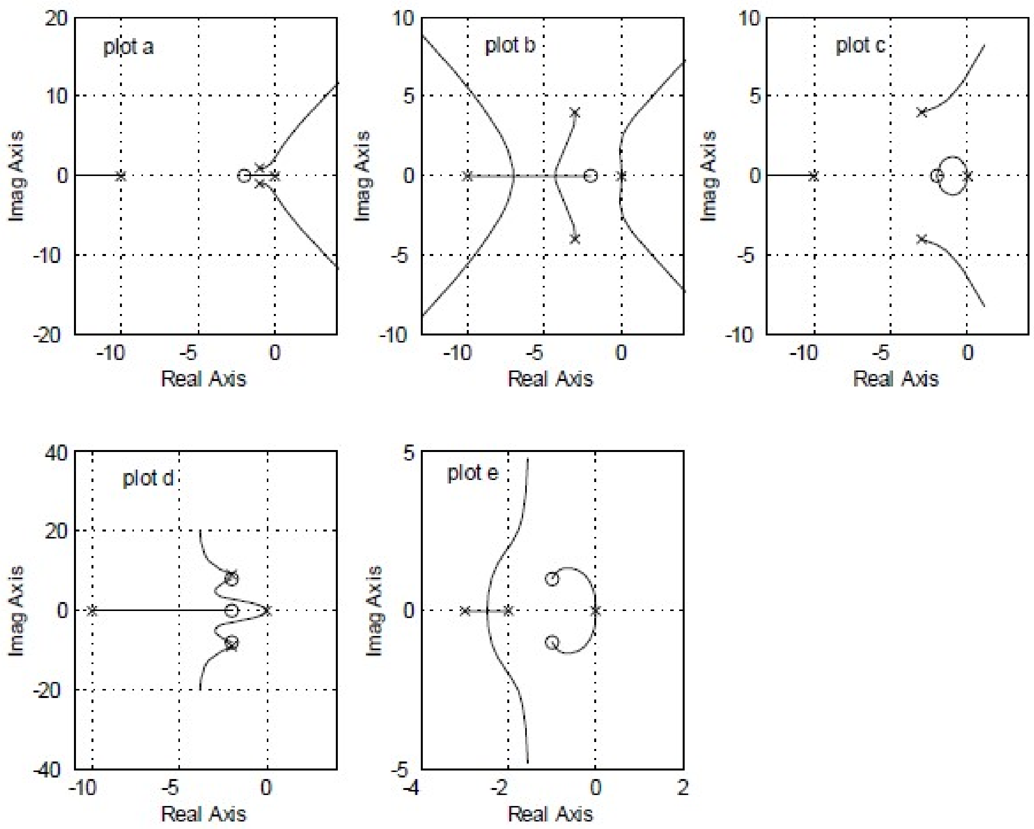
(b) $L(s) = \dfrac{(s+2)}{s^2(s+10)(s^2+6s+25)}$

(c) $L(s) = \dfrac{(s+2)^2}{s^2(s+10)(s^2+6s+25)}$

(d) $L(s) = \dfrac{(s+2)(s^2+4s+68)}{s^2(s+10)(s^2+4s+85)}$

(e) $L(s) = \dfrac{[(s+1)^2+1]}{s^2(s+2)(s+3)}$

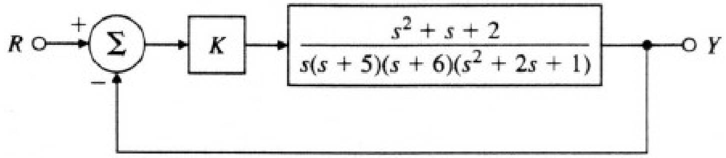
Exercício 8 - Solução



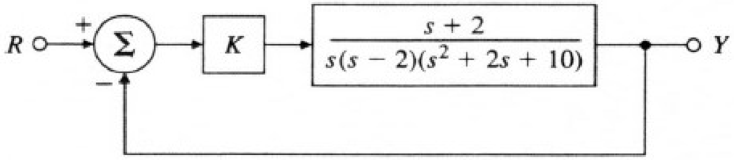
Solution for Problem 5.7

Exercício 9

11. Use the Routh criterion to find the range of the gain K for which the systems in Fig. 5.53 are stable and use the root locus to confirm your calculations.



(a)



(b)

Figure 5.53: Feedback systems for problem 11

Exercício 10

15. A simplified model of the longitudinal motion of a certain helicopter near hover has the transfer function

$$G(s) = \frac{9.8(s^2 - 0.5s + 6.3)}{(s + 0.66)(s^2 - 0.24s + 0.15)}.$$

and the characteristic equation $1 + D(s)G(s) = 0$. Let $D(s) = k_p$ at first.

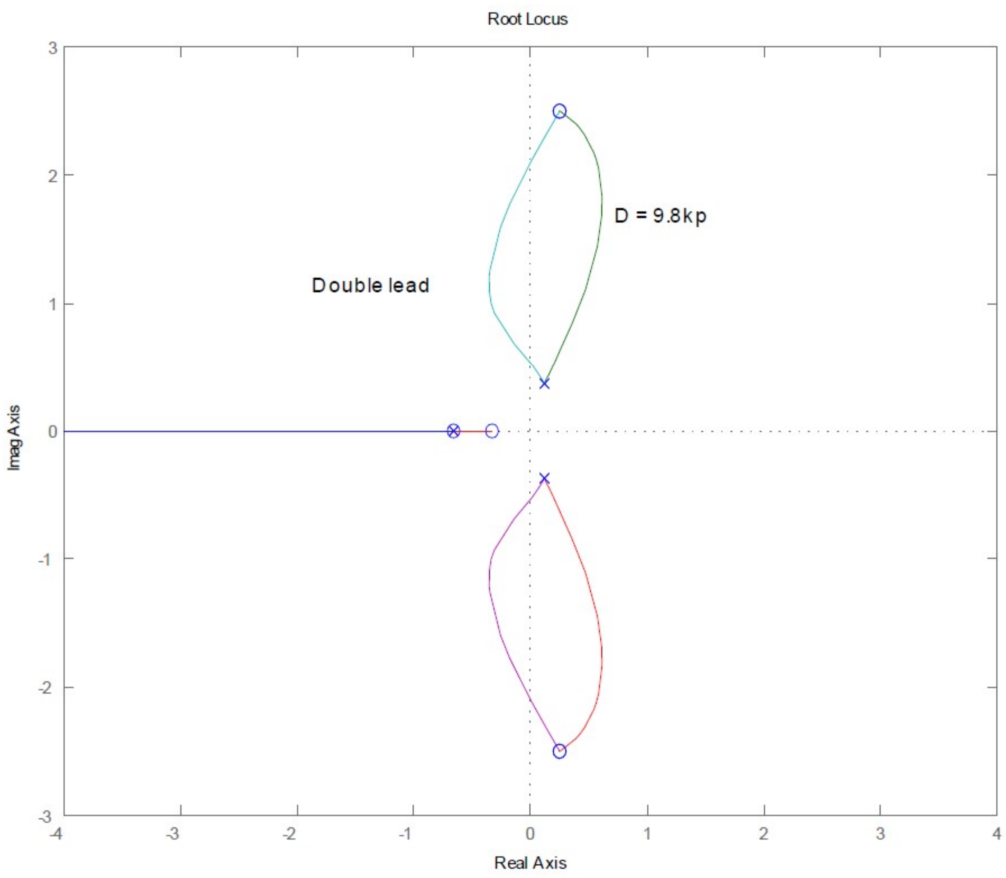
(a) Compute the departure and arrival angles at the complex poles and zeros.

(b) Sketch the root locus for this system for parameter $K = 9.8k_p$.Use axes -4 $\leq x \leq 4$. $-3 \leq y \leq 3$;

(c) Verify your answer using MATLAB. Use the command axis([-4 4 -3 3]) to get the right scales.

(d) Suggest a practical (at least as many poles as zeros) alternative compensation $D(s)$ which will at least result in a stable system.

Exercício 10 - LR



Exercício 11

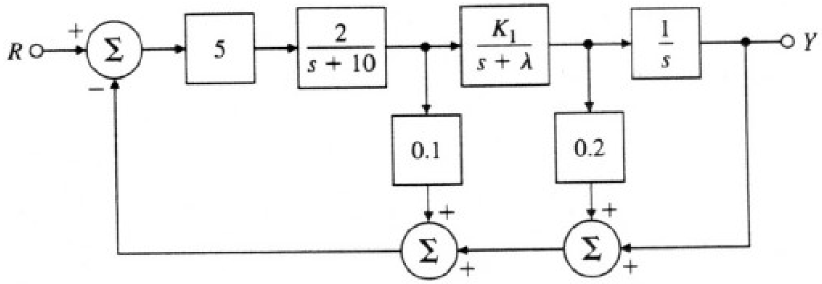


Figure 5.56: Control system for problem 5.16

16. For the system given in Fig. 5.56,

(a) plot the root locus of the characteristic equation as the parameter K_1 is varied from 0 to ∞ with $\lambda = 2$. Give the corresponding $L(s)$, $a(s)$, and $b(s)$.

(b) Repeat part (a) with $\lambda = 5$. Is there anything special about this value?

(c) Repeat part (a) for fixed $K_1 = 2$ with the parameter $K = \lambda$ varying from 0 to ∞.

Exercício 11

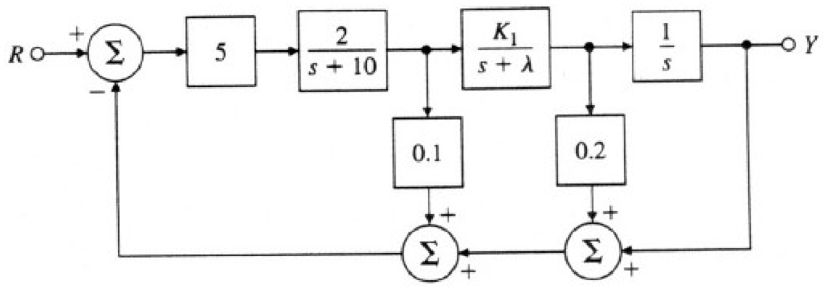


Figure 5.56: Control system for problem 5.16

16. For the system given in Fig. 5.56,

(a) plot the root locus of the characteristic equation as the parameter K_1 is varied from 0 to ∞ with $\lambda = 2$. Give the corresponding $L(s)$, $a(s)$, and $b(s)$.

(b) Repeat part (a) with $\lambda = 5$. Is there anything special about this value?

(c) Repeat part (a) for fixed $K_1 = 2$ with the parameter $K = \lambda$ varying from 0 to ∞.

13.43. Determine the angle and magnitude of

$$GH = \frac{20(s+10+j10)(s+10-j10)}{(s+10)(s+15)(s+25)}$$

at the following points in the s-plane: (*a*) $s = j10$, (*b*) $s = j20$, (*c*) $s = -10 + j20$, (*d*) $s = -20 + j20$, (*e*) $s = -15 + j5$.

13.44. For each transfer function, find the breakaway points on the root-locus:

(*a*) $GH = \dfrac{K}{s(s+6)(s+8)}$, (*b*) $GH = \dfrac{K(s+5)}{(s+2)(s+4)}$, (*c*) $GH = \dfrac{K(s+1)}{s^2(s+9)}$.

13.45. Find the departure angle of the root-locus from the pole at $s = -10 + j10$ for

$$GH = \frac{K(s+8)}{(s+14)(s+10+j10)(s+10-j10)} \qquad K > 0$$

13.46. Find the departure angle of the root-locus from the pole at $s = -15 + j9$ for

$$GH = \frac{K}{(s+5)(s+10)(s+15+j9)(s+15-j9)} \qquad K > 0$$

13.47. Find the arrival angle of the root-locus to the zero at $s = -7 + j5$ for

$$GH = \frac{K(s+7+j5)(s+7-j5)}{(s+3)(s+5)(s+10)} \qquad K > 0$$